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1. Perform the subtraction.

2. Perform the division.

3. Perform the addition, being sure to line up the decimal points.

4. $8\frac{3}{4} \div 3\frac{5}{6} = \frac{35}{4} \times \frac{6}{23} = \frac{35}{2} \times \frac{3}{23} = \frac{105}{46} = 2\frac{13}{46}$

5. $-4 - (-3)^{-2} = -4 - \frac{1}{9} = -\frac{37}{9}$

6. $975.31 = 9.7531 \times 10^2$

7. $2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 16 \times 8 = 80 + 48 = 128$

8. $\frac{6!}{3!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 6 \cdot 5 \cdot 4 = 30 \cdot 4 = 120$

9. $6 + (5 \times 4 - 3) \div 2 = 6 + (20 - 3) \div 2 = 6 + \frac{17}{2} = \frac{29}{2}$

10. $\sqrt[3]{400} = \sqrt[3]{2 \cdot 5 \cdot 2 \cdot 5 \cdot 2 \cdot 2} = 2\sqrt[3]{2 \cdot 5 \cdot 5} = 2\sqrt[3]{50}$

11. $987 \times 1013 = (1000 - 13)(1000 + 13) = 1000000 - 169 = 999831$

12. $60 \times 5 \times 100 = 300 \times 100 = 30000$

13. $19 \times 3 = 57$, before which $57 - 7 = 50$.

14. $4z + 5 = 33$ becomes $4z = 28$, so that $z = 7$.

15. $3y - 4 = 20 - y$ becomes $4y = 24$, so that $y = 6$.

16. $3x + 2 - x + 5x^2 - 7 + 9x - x^2 + 1 = 4x^2 + 11x - 4$

17. $w^2 - 8w + 15 = 0$ factors to $(w - 3)(w - 5) = 0$, with roots of 3 and 5.

18. The quadratic formula gives $v = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 2 \cdot 1}}{2 \cdot 2} = \frac{-5 \pm \sqrt{25 - 8}}{4} = \frac{-5 \pm \sqrt{17}}{4}$.

19. Substituting the first equation into the second gives $3x - 2(2x - 3) = 2$, then $3x - 4x + 6 = 2$, then $-x = -4$, so that $x = 4$. Substituting this into the first equation gives $y = 2 \cdot 4 - 3 = 8 - 3 = 5$.

20. The original job took five minutes. The number of mills was multiplied by $\frac{6}{2} = 3$, so the time must be divided by 3. The number of pounds was multiplied by $\frac{12}{3} = 4$, so the time must be multiplied by 4. This gives an answer of $\frac{5 \cdot 4}{3} = \frac{20}{3}$ minutes, which is $\frac{20}{3} \times 60 = 20 \cdot 20 = 400$ seconds.

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21. $3 \times 8 = 24$

22. The two numbers must each be $\frac{22}{2} = 11$ away from $\frac{58}{2} = 29$, so that they are $11 + 29 = 40$ and $29 - 11 = 18$, for an answer of 40.

23. The slope is $\frac{9-5}{2-4} = \frac{4}{-2} = -2$, so that the equation is $y = -2x + b$. Substituting the point $(4,5)$ gives $5 = -2 \times 4 + b = -8 + b$, so that $b = 13$ and the answer is $y = -2x + 13$.

24. The slope of this line is $-\frac{a}{b} = \frac{-3}{-5} = \frac{3}{5}$, and the slope of the perpendicular line is the negative reciprocal, $-\frac{5}{3}$.

25. The distance formula gives $\sqrt{(8-4)^2 + (-3-9)^2} = \sqrt{4^2 + 12^2} = 4\sqrt{1^2 + 3^2} = 4\sqrt{1+9} = 4\sqrt{10}$.

26. The point $(-1, -4)$ is six units to the left of the line $x = 5$, so the reflection will be six units to the right at $(11, -4)$.

27. The axis of symmetry will be $x = -\frac{b}{2a} = -\frac{24}{2 \cdot 3} = -\frac{24}{6} = -4$.

28. $y = x^2 + 4x - 12$ factors to $y = (x+6)(x-2)$, with zeros at $(-6,0)$ and $(2,0)$, with $(-6,0)$ being the leftmost one.

29. If the difference of the reversals is 18, the difference of the digits must be $\frac{18}{9} = 2$. The new number is the smaller one, so its smallest value is 13 (the reversal of 31).

30. We could already buy T pounds. We're multiplying the money by $\frac{P}{10D}$, so the amount of flour we can buy will be $\frac{TP}{10D}$.

31. There are either 1, 6, or 11 pennies. 11 pennies would need two coins to sum to 40 cents, which is impossible with dimes as our largest coin. 1 penny would need 12 coins to sum to 50 cents, which is impossible with nickels as our smallest coin. Thus, there are 6 pennies, leaving 7 coins that sum to 45 cents. If it were all dimes, it would be 70 cents, 25 cents too much. As we turn dimes into nickels, we lose five cents each time, so we need to do it five times, making our answer 5.

32. Although there are only two equations in three unknowns, there is enough information to solve for $r - 3u = 6$ and $s = 7$, so that $r + 2s - 3u = 6 + 2 \cdot 7 = 6 + 14 = 20$. An easier route would be to subtract the second equation from the first, getting the same result.

33. Cross-multiplying gives $6q^2 - 6q + 9q - 9 = 6q^2 + 15q - 2q - 5$, which becomes $3q - 9 = 13q - 5$, then $-4 = 10q$, for an answer of $q = -\frac{4}{10} = -\frac{2}{5}$.

34. $(2p+3)(p-2) = 2p^2 - 4p + 3p - 6 = 2p^2 - p - 6$

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35. The Pythagorean Theorem gives $\sqrt{11^2 - 7^2} = \sqrt{121 - 49} = \sqrt{72} = 6\sqrt{2}$.

36. There cannot be a 4-4-9 triangle, so it must be a 9-9-4 triangle. Half of this triangle is a right triangle with sides of 9, 2, and $\sqrt{9^2 - 2^2} = \sqrt{81 - 4} = \sqrt{77}$, making the original triangle's area $\frac{1}{2} \times 4 \times \sqrt{77} = 2\sqrt{77}$.

37. You just have to have memorized the term "equilateral". A proofer pointed out that "equiangular" and "regular" were also acceptable answers.

38. $3 + 8 + 3 + 8 = 11 + 11 = 22$

39. A diameter of 24 means a radius of $\frac{24}{2} = 12$, for an area of $\pi r^2 = \pi \times 12^2 = 144\pi$.

40. $8 \times 7 = 56$

41. $C = 2\pi r = 2 \times \pi \times 24 = 48\pi$

42. $SA = 2\pi r^2 + 2\pi rh = 2\pi r(r + h) = 2\pi \times 4(4 + 5) = 8\pi \times 9 = 72\pi$

43. If the cube's edges are 6, the sphere's radius is 3.
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi 3^3 = 4\pi 3^2 = 4\pi \times 9 = 36\pi$.

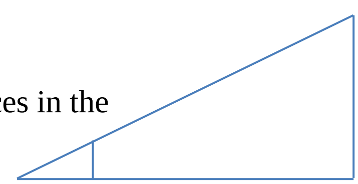
44. You could have this memorized as 120, or you could determine that the exterior angle is $\frac{360}{6} = 60$, so that the interior angle is $180 - 60 = 120$.

45. The ratio of the volumes is $\frac{54}{16} = \frac{27}{8}$, so the ratio of the dimensions is $\frac{27}{8}^{\frac{1}{3}} = \frac{3}{2}$ and the ratio of the areas is $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$. This makes the surface area of the larger prism $\frac{9}{4} \times 48 = 9 \times 12 = 108$.

46. $V = \frac{1}{3}Bh = \frac{1}{3}s^2h = \frac{1}{3} \times 5^2 \times 6 = 2 \times 25 = 50$

47. $V = \pi r^2 h = \pi \times 7^2 \times 8 = 49\pi \times 8 = 392\pi$

48. The figure to the right has two similar right triangles with distances in the ratio $\frac{38}{8} = \frac{19}{4}$, so that the streetlight is $\frac{19}{4} \times 5 = \frac{95}{4}$ feet tall.



49. $108 = \frac{73+x}{2}$, so that $216 = 73 + x$ and $x = 216 - 73 = 143$.

50. Angle bisectors break the opposite side into segments proportional to the lengths of the adjacent sides, so the smaller segment will measure $\frac{6}{6+8} \times 4 = \frac{24}{14} = \frac{12}{7}$.

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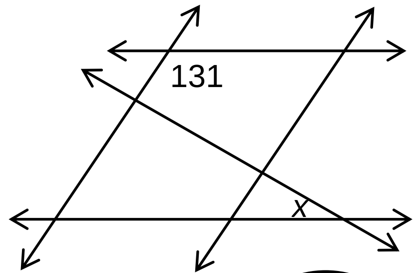
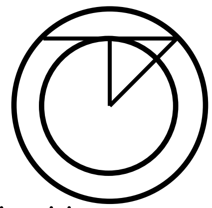


51. The small triangle can be shown to have angles of x, 90, and $180 - 131 = 49$, so that $x = 180 - 90 - 49 = 90 - 49 = 41$.



52. In the diagram to the right, half the chord has a length of $\sqrt{10^2 - 8^2} = \sqrt{100 - 64} = \sqrt{36} = 6$, for an answer of 12.

53. $4 + 4 + 3 + 2 + 1 + 1 = 4 + 10 + 1 = 15$

54. It may seem intuitive that the yardstick should form a 45-45-90 triangle. An intuitive way to prove this is best is to consider the shape formed by four yardsticks in four rectangular rooms sharing a corner; clearly their maximum area will be a square with an area of $3^2 = 9$ square feet. In this case, the area in one of the rooms is simply $\frac{9}{4}$.

55. The angle supplementary to 131 is $180 - 131 = 49$. $90 - 49 = 41$ is its complement, with $180 - 41 = 139$ being the supplement of 41.

56. The discriminant is $b^2 - 4ac = (-3)^2 - 4 \times 4(-2) = 9 + 32 > 0$ so there are **two** real roots.

57. $i^2 - 2i^3 + 3i^5 - 4i^7 + 5i^{11} = -1 + 2i + 3i + 4i - 5i = -1 + 4i$

58. Completing the square gives $(x-2)^2 + (y+3)^2 \cdots$, which is enough to know that the center is at the point $(2, -3)$.

59. If the points were equidistant from the point and line, they'd be a parabola. These points are closer to the line than a parabola would be, so they're a **hyperbola**. Points farther from the line than the point would form an ellipse.

60. Much like a circle, $A = \pi \times r_1 \times r_2 = \pi \times 5 \times 6 = 30\pi$.

61. $k(-3) = 3(-3)^3 - 7(-3) + 1 = -81 + 21 + 1 = -60 + 1 = -59$

62. If $h(g) = 3g - 4$, then $g = 3h^{-1}(g) - 4$, so $29 = 3h^{-1}(29) - 4$, which becomes $33 = 3h^{-1}(29)$, giving $h^{-1}(29) = \frac{33}{3} = 11$.

63. You could solve for the roots, square them, and then add, but it would be messy and time-consuming (and easy to make a mistake). Using the fact that the sum of the roots is $f_1 + f_2 = -\frac{b}{a} = -\frac{-7}{3} = \frac{7}{3}$ and the product of the roots is $f_1 f_2 = \frac{c}{a} = \frac{11}{3}$, we can write $f_1^2 + f_2^2 = (f_1 + f_2)^2 - 2f_1 f_2 = \left(\frac{7}{3}\right)^2 - 2 \times \frac{11}{3} = \frac{49}{9} - \frac{22}{3} = \frac{49}{9} - \frac{66}{9} = -\frac{17}{9}$.

64. James had the correct value of Z, which must have been $-(1 \times 2 \times 3) = -6$. Cherie had the correct value of Y, which must have been $2 \times 3 + 3 \times 4 + 2 \times 4 = 6 + 12 + 8 = 26$. Tom had the correct value of X, which must have been $-(3 + 4 + 5) = -12$. The sum of these is $-12 + 26 - 6 = 26 - 18 = 8$.

65. In base six the digits from right to left represent $6^0 = 1$'s, $6^1 = 6$'s, $6^2 = 36$'s, $6^3 = 216$'s, $6^4 = 1296$'s, etc. There are no 1296's in 648, but there are three 216's, which total 648, so there are no 36's, 6's, or 1's left, for an answer of 3000.

66. Every digit in base four is the equivalent of two digits in base two, because $2^2 = 4$. Thus, we don't need to convert the entire number, just each digit. $2_4 = 10_2$, $2_4 = 10_2$, and $3_4 = 11_2$, for an answer of 111010_2.

67. $540 = 2 \times 5 \times 2 \times 27 = 2^2 \times 3^3 \times 5$

68. $160 = 2^4 \times 2 \times 5 = 2^5 \times 5$ A factor of 160 can have from zero to five 2's in its prime factorization (six choices) and from zero to one 5 (two choices), for an answer of $6 \times 2 = 12$.

69. The greatest common factor must also be a factor of the difference, $138 - 126 = 12$. 12 is clearly not a factor of 126, so 6 is the answer.

70. $\frac{4}{4+12} = \frac{4}{16} = \frac{1}{4}$

71. Any first card will work, so the question comes down to the probability that the second card matches the first. There are three of the chosen rank left, out of 51 remaining cards, for a probability of $\frac{3}{51} = \frac{1}{17}$.

72. There are six ways to get a sum of 7 (the most likely roll), five ways to get a sum of 6, four ways to get a sum of 5, etc. $\frac{4}{36} = \frac{1}{9}$.

73. The five letters in EVENT can be arranged in $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ ways. But because two of the letters are the same (the E's), we need to divide by the $2! = 2 \times 1 = 2$ ways the E's could be arranged in any particular arrangement of the five letters, because those two arrangements of the E's are indistinguishable. $\frac{120}{2} = 60$.

74. There are $4! = 4 \times 3 \times 2 \times 1 = 24$ ways to arrange the publishers, $2! = 2$ ways to arrange the AAA books, $3! = 6$ ways for Lonely Planet, $1! = 1$ for Moon, and $2! = 2$ for Frommers, for a total of $24 \times 2 \times 6 \times 1 \times 2 = 24 \times 24 = 576$ ways.

75. The first point doesn't matter. The second point can be up to 1/3 of the circle away in either direction, for a probability of $\frac{2}{3}$.

76. The differences in the sequence are 3, 5, ?, ?(total of 16), 11, 13, ?, ?(total of 32). The differences appear to be the odd numbers, and in fact $7 + 9 = 16$ and $15 + 17 = 32$. This makes the missing terms $22 + 7 = 29$ and $62 + 15 = 77$, for an answer of $29 + 77 = 106$.

77. This sequence is actually two interspersed sequences: 1, 2, 4, 8, (16), 32 and 4, (7), 10, 13, 16. $16 + 7 = 23$.

78. The two-digit multiples of four go from 12 to 96, which are four times the numbers from 3 to 24. Thus, there are $24 - 3 + 1 = 22$ numbers. If we add the first and last numbers, we get $12 + 96 = 108$. If we add the second and penultimate numbers, we get $16 + 92 = 108$ again. There will be $\frac{22}{2} = 11$ such "outer pairs", each with a sum of 108, for an answer of $11 \times 108 = 1188$.

79. The differences of this sequence are 3, 9, 27, ?, ?(sum of 324), and 729, which appear to be powers of 3, and $81 + 243 = 324$, so the answer is $43 + 81 = 124$.

80. The numbers from 2 to 30 make $\frac{15}{2}$ outer pairs with sums of $2 + 30 = 32$, for a total of $\frac{15}{2} \times 32 = 15 \times 16 = 240$.

81. The set is so small that it is probably easier to list it than to do inclusion/exclusion. The numbers in question are $1 + 2 + 5 + 7 + 10 + 11 + 13 + 14 + 17 + 19 = 30 + 30 + 20 + 10 + 9 = 99$.

82. The median is the middle number when they're arranged in order, so 5.

83. The numbers in order must be *x*, *y*, 50, 60, 60, with $x \neq y$. The sum is $x + y + 170 = 5 \times 40 = 200$, so that $x + y = 30$. For a small range, we'd like *x* to be as high as possible, but less than *y*, so $x = 14$ and the range is $60 - 14 = 46$.

84. Depending on the values of *b* and *c*, the median could be anything from 6 to 9, the unique mode could be anything 7 or larger, and the mean could be anything over 7. The mean is currently just over 7 $\left(\frac{2+6+8+9+11}{5} = \frac{36}{5}\right)$, so *b* & *c* must be larger than 7, so now the median can only be 8 or 9, the unique mode must be 9 or higher, and the mean must be bigger than 9. If the unique mode is 9 or higher, then the median must be 9, the unique mode is 10 or higher, and the mean is higher than 10. If the mean is higher than 10, then the mode is not 10 and must be 11 (*b*), with the mean higher than 11. The smallest possible value of *c* must satisfy $2 + 6 + 8 + 9 + 11 + 11 + c = 47 + c > 7 \times 11 = 77$, so that $c > 30$ and $c = 31$. This makes $b + c = 11 + 31 = 42$.

85. There are $\frac{69}{3} = 23$ multiples of three less than 70, but that includes 30, 45, and 60, making the answer $23 - 3 = 20$.

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86. The numbers 4, 8, and 16 are in both sets. All of our subsets are made from these numbers, and each subset can either have or NOT have each of those three numbers. For example, Subset Q can have a 4, but subset R can NOT have a 4. Thus, when we're building a subset, we have two choices for whether or not to put 4 in it, two choices for 8, and two choices for 16, for a total of $2 \times 2 \times 2 = 8$ choices. These choices make all the possible subsets, so our answer it 8.

87. There are $5 \times 9 = 45$ 1x1 squares, $4 \times 8 = 32$ 2x2 squares, $3 \times 7 = 21$ 3x3's, $2 \times 6 = 12$ 4x4's, and $1 \times 5 = 5$ 5x5's, for a total of $45 + 32 + 21 + 12 + 5 = 115$ squares of any size.

88. Trial and error hopefully led you quickly to $6 \times 7 - 4 = 38$.

89. $\begin{vmatrix} 1 & 2 \\ -3 & 4 \end{vmatrix} = 1 \times 4 - (-3) \times 2 = 4 + 6 = 10$

90. $5(-2) + (-6)(-4) = -10 + 24 = 14$

91. The Law of Sines gives $\frac{PR}{\sin 45} = \frac{36}{\sin 60}$, which becomes $\frac{PR}{\frac{\sqrt{2}}{2}} = \frac{36}{\frac{\sqrt{3}}{2}}$, then $PR = \frac{36\sqrt{2}}{\sqrt{3}} = \frac{36\sqrt{6}}{3} = 12\sqrt{6}$.

92. $\frac{3\pi}{5} \times \frac{180}{\pi} = \frac{3 \times 180}{5} = 3 \times 36 = 108$

93. $\sin^2 b + \sec^2 b - 1 + \cos^2 b = 1 + \sec^2 b - 1 = \sec^2 b = \frac{1}{\cos^2 b} = \tan^2 b + 1$

94. The graph of $r = \sin\theta$ is a circle with a diameter from (0,0) to $\left(\frac{\pi}{2}, 1\right)$. This circle has a diameter of 1, so a radius of $\frac{1}{2}$ and an area of $\times \left(\frac{1}{2}\right)^2 = \frac{\pi}{4}$.

95. $\lim_{w \to 0} \frac{y(2) - y(2+2w)}{w}$ is the definition of the derivative at $x = 2$, except that the subtraction is in the wrong order and there is a separation of $2w$ instead of just w. Thus, our answer will be negative two times the derivative at $x = 2$. The derivative at $x = 2$ is $2 \times 2 + 1 = 4 + 1 = 5$, so our answer is $-2 \times 5 = -10$.

96. $u'(v) = \ln(2v) + v \times \frac{1}{2v} \times 2 = \ln(2v) + 1$, so $u'(e) = \ln(2e) + 1 = \ln 2 + \ln e + 1 = \ln 2 + 1 + 1 = \ln 2 + 2$.

97. To be continuous, the two parts of the function need to meet when $t = 3$. The two values are $2 \times 3^2 + 3 = 2 \times 9 + 3 = 18 + 3 = 21$ and $4 \times 3 + r = 12 + r$. Setting them equal gives $21 = 12 + r$, which leads to $r = 9$.

98. If $d(t) = 4t^3 - 3t + 2$, $v(t) = 3 \times 4t^2 - 3$ and $a(t) = 2 \times 3 \times 4t = 24t$, so $a(5) = 24 \times 5 = 120$.

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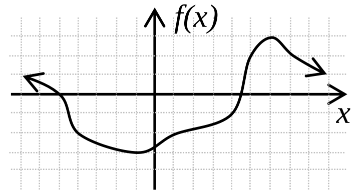

99. $f'(x) > 0$ means that the value of *f* is increasing, which happens from $x = -1$ and $x = 6$, for an answer of $(-1,6)$.

100. $\int_0^5 |x-2|\,dx$ is the area under the absolute value function with a minimum at $x = 2$, which will be two isosceles right triangles with legs measuring 2 and 3, for an answer of $\frac{1}{2} \times 2^2 + \frac{1}{2} \times 3^2 = \frac{4}{2} + \frac{9}{2} = \frac{13}{2}$.