

2012 Fall Startup Event  
Thursday, September 27th, 2012

This test consists of 100 problems to be solved in 30 minutes. All answers must be exact, complete, and in simplest form. **To ensure consistent grading, if you get a decimal, mixed number, or ratio as any part of an answer, it should be expressed as a fraction unless otherwise specified in the problem.** A correct answer to a problem scores one point; a blank or incorrect answer to a problem scores no points. All answers must be written on the answer sheet in the boxes provided; work or answers written elsewhere will not be scored.

1. Evaluate:  $4719 - 1833$
2. Evaluate:  $1044 \div 18$
3. Evaluate **as a decimal**:  $234.56 + 98.7$
4. Evaluate **as a mixed number**:  $8\frac{3}{4} \div 3\frac{5}{6}$
5. Evaluate:  $-4 - (-3)^{-2}$
6. Express the number 975.31 in scientific notation.
7. Evaluate:  $2^7$
8. Evaluate:  $\frac{6!}{3!}$
9. Evaluate:  $6 + (5 \times 4 - 3) \div 2$
10. Express in simplest radical form:  $\sqrt[3]{400}$
11. Evaluate:  $987 \times 1013$
12. What is the product of the number of minutes in an hour, the number of sides on a pentagon, and the number of centimeters in a meter?
13. When my secret number is increased by seven and this result is divided by three, the final result is nineteen. What is my secret number?
14. What value(s) of  $z$  satisfy  $4z + 5 = 33$ ?
15. What value(s) of  $y$  satisfy  $3y - 4 = 20 - y$ ?
16. Simplify by combining like terms:  $3x + 2 - x + 5x^2 - 7 + 9x - x^2 + 1$
17. What value(s) of  $w$  satisfy  $w^2 - 8w + 15 = 0$ ?
18. What value(s) of  $v$  satisfy  $2v^2 + 5v + 1 = 0$ ?
19. What are the coordinates, in the form  $(x, y)$ , of the point of intersection of the lines  $y = 2x - 3$  and  $3x - 2y = 2$ ?

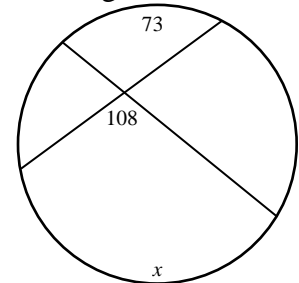
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20. If two mills can grind three pounds of flour in five minutes, how many **seconds** would it take six mills to grind twelve pounds of flour?
21. If JT rides his bike at eight miles per hour for three hours, how many miles will he ride?
22. If two numbers sum to 58 and differ by 22, what is the value of the larger number?
23. What is the equation, in slope-intercept form ( $y = mx + b$ ), of the line through the points (4,5) and (2,9)?
24. What is the slope of a line perpendicular to the line with equation  $3x - 5y = 7$ ?
25. What is the distance between the points (8, -3) and (4,9)?
26. What are the coordinates, in the form (x, y), of the reflection of (-1, -4) across the line  $x = 5$ ?
27. What is the equation of the axis of symmetry of the parabola  $y = 3x^2 + 24x - 7$ ?
28. What are the coordinates of the leftmost x-intercept of the graph of  $y = x^2 + 4x - 12$ ?
29. When the digits of a positive two-digit integer are reversed to form a new positive two-digit integer, the result is 18 less than the original number. What is the smallest possible value of the new number?
30. If you can buy T pounds of turnips with D dimes, how many pounds of turnips could you buy with P pennies?
31. If my coin purse contains thirteen coins worth a total of 51 cents, how many nickels are there?
32. In the system of equations  $2r + s - 6u = 19$  and  $r - s - 3u = -1$ , what is the value of  $r + 2s - 3u$ ?
33. What value(s) of  $q$  satisfy  $\frac{q-1}{2q+5} = \frac{3q-1}{6q+9}$ ?
34. Simplify by multiplying and combining like terms:  $(2p + 3)(p - 2)$
35. What is the length, in meters, of the other leg of a right triangle with a hypotenuse measuring eleven meters and one leg measuring seven meters?
36. What is the area, in square meters, of an isosceles triangle with sides measuring nine meters and four meters?
37. What is the name for a triangle with three congruent sides?
38. What is the perimeter, in meters, of a rectangle with sides measuring three meters and eight meters?

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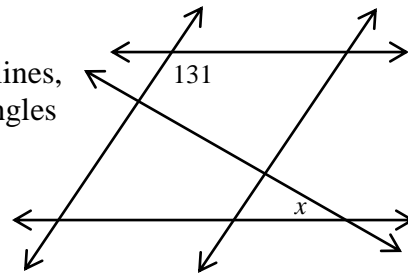
39. What is the area, in square meters, of a circle with a diameter measuring 24 m?
40. What is the perimeter, in meters, of an octagon with sides measuring seven meters?
41. What is the circumference, in meters, of a circle with a radius measuring 24 m?
42. What is the surface area, in square meters, of a right circular cylinder with a height of five meters and a base radius of four meters?
43. What is the volume, in cubic meters, of a sphere inscribed in a cube with edges measuring six meters?
44. What is the measure, in degrees, of an interior angle of a regular hexagon?
45. Two right rectangular prisms are similar to one another with volumes of  $16 \text{ m}^3$  and  $54 \text{ m}^3$ . If the smaller of the two has a surface area of  $48 \text{ m}^2$ , what is the surface area of the larger prism, in square meters?
46. What is the volume, in cubic meters, of a right square pyramid with base edges measuring five meters and a height of six meters?
47. What is the volume, in cubic meters, of a right circular cylinder with a base radius of seven meters and a height measuring eight meters?
48. When Nathan stands thirty feet from a streetlight, his shadow is eight feet long. If Nathan is five feet tall, how many feet tall is the streetlight?

49. In the circle shown with intersecting chords, angles and arcs are measured in degrees. What is the value of  $x$ ?



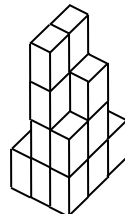
50. A triangle has sides measuring four meters, six meters, and eight meters. If a line is drawn bisecting the smallest angle, what is the length in meters of the smaller of the two segments into which the opposite side is divided?

51. The figure to the right contains two pairs of parallel lines, one line perpendicular to one of the pairs, and two angles labeled in degrees. What is the value of  $x$ ?



52. Two circles have radii of eight meters and ten meters, and their centers are concurrent. What is the length, in meters, of a chord of the larger circle that is tangent to the smaller circle?

53. How many blocks are in the stack shown?



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54. A yardstick is to be laid on the floor in the corner of a rectangular room. What is the largest floor area, in **square feet**, that can be contained between the yardstick and the two walls?
55. What is the measure, in degrees, of an angle supplementary to an angle complementary to an angle supplementary to an angle of  $131^\circ$ ?
56. How many distinct real roots does the equation  $4m^2 - 3m - 2 = 0$  have?
57. Simplify, where  $i = \sqrt{-1}$ :  $i^2 - 2i^3 + 3i^5 - 4i^7 + 5i^{11}$
58. What are the coordinates, in the form  $(x, y)$ , of the center of the circle with equation  $x^2 + y^2 - 4x + 6y - 100 = 0$ ?
59. What is the name for the shape of the locus of points that is half as far from the line  $y = 2x - 3$  as from the point  $(5, -2)$ ?
60. What is the area, in square meters, of an ellipse with axes measuring ten meters and twelve meters?
61. If  $k(j) = 3j^3 - 7j + 1$ , evaluate  $k(-3)$ .
62. If  $h(g) = 3g - 4$ , evaluate  $h^{-1}(29)$ .
63. What is the sum of the squares of the roots of the equation  $3f^2 - 7f + 11 = 0$ ?
64. When Mr. Norris puts a cubic equation of the form  $d^3 +Xd^2 +Yd +Z = 0$  on the board, James writes incorrect values of  $X$  &  $Y$ , getting roots of 1, 2, and 3. Cherie writes incorrect values of  $X$  &  $Z$ , getting roots of 2, 3, and 4. Tom writes incorrect values of  $Y$  &  $Z$ , getting roots of 3, 4, and 5. What was the sum of  $X$ ,  $Y$ , and  $Z$  in Mr. Norris's original equation?
65. Express the base ten number  $648_{10}$  as a base six number.
66. Express the base four number  $322_4$  as a binary number.
67. What is the prime factorization of 540 in exponential form?
68. How many positive integers are factors of 160?
69. What is the greatest common factor of 126 and 138?
70. When a marble is drawn from a bag containing four red marbles and twelve blue marbles, what is the probability that it is red?
71. When two cards are drawn from a standard 52-card deck, what is the probability that they form a pair (have the same rank, e.g. they are both sevens)?
72. When two fair six-sided dice are rolled, what is the probability that they show a sum of five?
73. In how many unique ways can the letters in the word EVENT be arranged?

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74. In my travel library I have two “AAA” guidebooks, three from “Lonely Planet”, one from “Moon”, and two from “Frommers”. In how many ways can I arrange these next to one another on a shelf if I must keep books from the same publisher together?
75. Two points are picked on a circular ring and the ring is cut at these points. What is the probability that one of the resulting pieces is more than twice as long as the other?
76. What is the sum of the missing terms of the sequence 14, 17, 22, \_\_, 38, 49, 62, \_\_, 94?
77. What is the sum of the missing terms of the sequence 1, 4, 2, \_\_, 4, 10, 8, 13, \_\_, 16, 32?
78. What is the sum of all positive two-digit multiples of four?
79. What is the missing term of the sequence 4, 7, 16, 43, \_\_, 367, 1096?
80. What is the sum of the fifteen smallest positive even numbers?
81. What is the sum of the positive integers less than twenty that are neither multiples of three nor four?
82. What is the median of the data set {3, 7, 4, 5, 12}?
83. In a five-element set of integer test scores from 0 to 100 inclusive, the mean is 40, the median is 50, and the unique mode is 60. What is the smallest possible value of the range?
84. In the set of positive integers {2, 6, 8, 9, 11,  $b$ ,  $c$ }, the mean is larger than the unique mode, which is larger than the median. What is the smallest possible sum of  $b$  and  $c$ ?
85. If Set A is the set of all positive multiples of three less than 70 and Set Z is the set of all positive multiples of five between 20 and 200 inclusive, how many elements are in the set  $A \cap Z'$ ?
86. If Set Y is the set of positive composite numbers less than 30 and Set X is the set of all positive powers of two less than 30, how many subsets of Set Y are also subsets of Set X?
87. How many squares of any size appear in a five-by-nine grid of unit squares?
88. Using the digits four, six, and seven exactly once each, and the operations of addition, subtraction, multiplication, and division (and parentheses) as many times as you like, create an expression that evaluates to 38.
89. Evaluate the determinant:  $\begin{vmatrix} 1 & 2 \\ -3 & 4 \end{vmatrix}$
90. Evaluate the dot product:  $\langle 5, -6 \rangle \cdot \langle -2, -4 \rangle$
91. In  $\triangle PQR$ ,  $PQ = 36$  m,  $m\angle Q = 45^\circ$ , and  $m\angle R = 60^\circ$ . What is the length, in meters, of  $\overline{PR}$ ?
92. Convert  $\frac{3\pi}{5}$  radians to degrees.

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93. Simplify:  $\sin^2 b + \sec^2 b - 1 + \cos^2 b$

94. What is the area of the region enclosed by the polar graph of  $r = \sin \theta$ ?

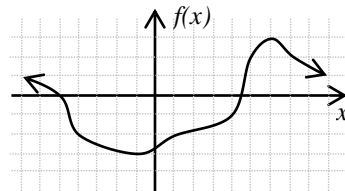
95. If  $y(x) = x^2 + x$ , evaluate  $\lim_{w \rightarrow 0} \frac{y(2) - y(2+2w)}{w}$ .

96. If  $u(v) = v \ln(2v)$ , evaluate  $u'(e)$ .

97. If  $s(t) = 2t^2 + t$  for  $t < 3$  and  $s(t) = 4t + r$  for  $t \geq 3$ , what value of  $r$  will make  $s$  a continuous function of  $t$ ?

98. If the distance from a point on a line to a particle moving along that line is given by  $d(t) = 4t^3 - 3t + 2$ , what is the acceleration of the particle at time  $t = 5$ ?

99. For the graph of  $f(x)$  shown to the right on a unit grid, express in interval notation the values of  $x$  for which  $f'(x) > 0$ .



100. Evaluate:  $\int_0^5 |x - 2| dx$