2012 Team Scramble
Solutions

1. **Evaluate: $\mathbf{1234+567+89012+34}$**

   Just do the addition: $1234+567+89012+34=90847$

2. **What number is eight more than twice the product of nine and the sum of ten and eleven?**

   $2\times 9(10+11)+8=18\times 21+8=378+8=386$

3. **What percent of 180 gives 27?**

   $\frac{x}{100}\times 180=27$ becomes $\frac{9x}{5}=27$ and gives $x=27\times\frac{5}{9}=3\times 5=15$.

4. **Evaluate: $\mathbf{-3-(-2)^{-1}(-9)}$**

   $$-3-(-2)^{-1}(-9)=-3-\frac{9}{2}=-\frac{15}{2}$$

5. **Evaluate: $\frac{\mathbf{4!\times 7!}}{\mathbf{6!\times 5!}}$**

   $$\frac{4!\times 7!}{6!\times 5!}=\frac{7}{5}$$

6. **Express in simplest radical form: $\sqrt{\frac{\mathbf{12\times 18\times 20\times 32}}{\mathbf{15\times 6}}}$**

   $$\sqrt{\frac{12\times 18\times 20\times 32}{15\times 6}}=\sqrt{12\times 4\times 32}=4\times 4\sqrt{3\times 2}=16\sqrt{6}$$

7. **Evaluate: $\mathbf{41^2-39^2}$**

   $41^2-39^2=(41-39)(41+39)=2\times 80=160$

8. **Arrange the variables A-E in increasing order (e.g. your answer might be ABCDE).**
   $\boldsymbol{A=\binom{100}{97}}$ $\boldsymbol{B=\sqrt{200{,}000}}$ $\boldsymbol{C=2.3^{45}}$ $\boldsymbol{D=98\times 76}$ $\boldsymbol{E=8!}$

   $A=\frac{100\times 99\times 98}{3\times 2}=100\times 33\times 49\sim 3300\times\frac{100}{2}=165{,}000$, $B=100\sqrt{20}=200\sqrt{5}\sim 446$, $C>2^{45}>1024^4\sim 1{,}000{,}000{,}000{,}000$, $D\sim 7600$, $E=8\times 7\times 720\sim 8\times 5000=40{,}000$. Thus, the proper order is BDEAC.

9. **When my secret number is decreased by thirteen, this result is tripled, and that result is increased by thirty, the final result is 2325. What is my secret number?**

   $(S-13)\times 3+30=2325$ becomes $3S-39+30=2325$, then $3S-9=2325$, then $3S=2334$, finally giving $S=778$.

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10. What value(s) of *f* satisfy $97f + 89 = 83 - 79f$?

$97f + 89 = 83 - 79f$ becomes $176f = -6$, giving $f = -\frac{6}{176} = -\frac{3}{88}$.

11. What value(s) of *g* satisfy $6(5g - 4) + 3 = 2 - 9(8 + 7g)$?

$6(5g - 4) + 3 = 2 - 9(8 + 7g)$ **becomes** $30g - 24 + 3 = 2 - 72 - 63g$, then $93g = -49$, finally giving $g = -\frac{49}{93}$.

12. What value(s) of *h* satisfy $12h^2 - 88h - 15 = 0$?

Factoring gives $(2h - 15)(6h + 1) = 0$, for roots of $\frac{15}{2}$ and $-\frac{1}{6}$.

13. The sum of two numbers is 987 and their difference is 567. What is the smaller of the two numbers?

The smaller number will be $\frac{987-567}{2} = \frac{420}{2} = 210$.

14. Express the equation of the line through the point $(3, 5)$ and parallel to the line $3x + y = 7$ in slope-intercept $(y = mx + b)$ form.

The parallel line will be of the form $3x + y = b$. Substituting gives $b = 3 \times 3 + 5 = 9 + 5 = 14$, so that the line is $3x + y = 14$, which in slope-intercept form becomes $y = -3x + 14$.

15. What is the shortest distance from the point $(-2, 9)$ to the line $y = 7x + 4$?

The shortest distance from a point $(j, k)$to a line in the form $Ax + By + C = 0$ is $\frac{|Aj+Bk+C|}{\sqrt{A^2+B^2}} = \frac{|-7(-2)+9-4|}{\sqrt{7^2+1^2}} = \frac{|14+5|}{\sqrt{50}} = \frac{19}{5\sqrt{2}} = \frac{19\sqrt{2}}{10}$.

16. What are the coordinates, in the form (x, y), of the vertex of the parabola $y = 2x^2 - 6x + 7$?

The axis of symmetry will be $x = -\frac{b}{2a} = -\frac{-6}{2\times2} = \frac{6}{4} = \frac{3}{2}$, so that the y-coordinate of the vertex will be $y = 2\left(\frac{3}{2}\right)^2 - 6\left(\frac{3}{2}\right) + 7 = \frac{9}{2} - \frac{18}{2} + 7 = -\frac{9}{2} + 7 = \frac{5}{2}$.

17. What is the smallest possible positive difference between a positive four-digit integer and the positive four-digit integer formed by reversing its digits?

If we start with ABCD, the reversal is DCBA, and the difference will be $999(A - D) + 90(B - C)$. This will be a minimum when $A - D = 0$ and $B - C = 1$, for an answer of 90.

18. At the Longneck Ranch, a corral contains Emus & Llamas. If there are 44 heads and 144 feet, how many Emus are in the corral?

Assuming it's all Llamas, 44 heads should mean $4 \times 44 = 176$ feet, which is $176 - 144 = 32$ feet too many. For each Llama we swap out for an Emu, we lose $4 - 2 = 2$ feet, so we'll need $\frac{32}{2} = 16$ of them.

19. In a set of three numbers, the sums of the two-element subsets are 14, 31, and 9. What is the value of the smallest number?

We can write $a + b = 14$, $b + c = 31$, and $a + c = 9$. Adding the three gives $2a + 2b + 2c = 54$, which becomes $a + b + c = 27$. Subtracting the second equation from this gives $a = 27 - 31 = -4$.

20. Two circles have radii of eight meters and ten meters, and their centers are twelve meters apart. What is the length, in meters, of one of their common external tangents?

Drawing the circles, the tangent, radii to the tangent, a segment between the centers and a segment parallel to the tangent ending on the center of the smaller circle, we can form a right triangle with a hypotenuse of 12, a leg of $10 - 8 = 2$, and another leg congruent to the tangent. This makes the tangent length $\sqrt{12^2 - 2^2} = 2\sqrt{35}$.

21. What is the perimeter, in meters, of an equilateral triangle with an area of $96\sqrt{3}$ m^2?

Using $A = \frac{s^2\sqrt{3}}{4}$, we can write $96\sqrt{3} = \frac{s^2\sqrt{3}}{4}$, which becomes $4 \times 96 = s^2$, giving $s = \sqrt{4 \times 96} = 2 \times 4\sqrt{6} = 8\sqrt{6}$. This makes the perimeter $3 \times 8\sqrt{6} = 24\sqrt{6}$.

22. What is the name of a triangle where each angle measures less than $90°$?

Each angle is "acute", so the triangle is called "acute".

23. What is the perimeter, in meters, of a convex polygon with sides measuring 9 m each where the number of sides of the polygon equals its number of diagonals?

We can write $n = \frac{n(n-3)}{2}$, which becomes $2n = n^2 - 3n$, then $0 = n^2 - 5n = n(n - 5)$ with roots of 0 and 5. I'm unfamiliar with the 0-gon, so we must have a pentagon with 9m sides, for a perimeter of $9 \times 5 = 45$.

24. What is the area, in square meters, of an isosceles right triangle inscribed in a circle with a radius of 8 m?

Any right triangle in a circle has its hypotenuse as a diameter of the circle, thus its hypotenuse is $2 \times 8 = 16$. Each leg is thus $\frac{16}{\sqrt{2}} = \frac{16\sqrt{2}}{2} = 8\sqrt{2}$, for an area of $\frac{1}{2}\left(8\sqrt{2}\right)^2 = 64$.

25. What is the general name for a polygon with four sides?

You just had to have this memorized: "quadrilateral".

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26. A rhombus with a perimeter of 12 m and an area of 8 m^2 is similar to another rhombus with a perimeter of 16 m. What is the area, in square meters, of the larger rhombus?

You could figure out the shape of the smaller rhombus and then use that same shape for the larger one to find its area, but it would be overkill. Two similar shapes have ratios that you can use to solve the problem: if the distances have a ratio of r, the areas have a ratio of r^2 and the volumes have a ratio of r^3. Thus, because the perimeter ratio is $\frac{16}{12} = \frac{4}{3}$, the area ratio will be $\left(\frac{4}{3}\right)^2 = \frac{16}{9}$, so that the area of the larger rhombus will be $8 \times \frac{16}{9} = \frac{128}{9}$.

27. How many edges does a regular dodecahedron have?

A dodecahedron has twelve sides, each of which is a pentagon. This is a total of $12 \times 5 = 60$ edges, but because each edge is counted twice this way we must divide by two to get $\frac{60}{2} = 30$.

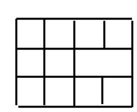
28. In a triangle with sides measuring 7 m, 9 m, and 12 m, what is the length, in meters, of the altitude to the shortest side?

The area of the triangle is $\sqrt{14 \times 7 \times 5 \times 2} = 14\sqrt{5}$. The altitude to the shortest (7) side is $h = \frac{14\sqrt{5}}{\frac{7}{2}} = 4\sqrt{5}$.

29. A cow is tied to an external corner of a rectangular barn with sides measuring 20 m by 40 m. If the length of the cow's rope is 50 m, what is the area, in square meters, of the region in which the cow can graze?

The cow can graze three-quarters of a 50m circle and one-quarter of both a 30m circle and a 10m circle, for an area of $\frac{3}{4}\pi \times 50^2 + \frac{1}{4}\pi \times 30^2 + \frac{1}{4}\pi \times 10^2 = \frac{\pi}{4}(7500 + 900 + 100) = \frac{8500\pi}{4} = 2125\pi$.

30. How many squares of any size are there in the array of unit squares shown with one missing segment?



There are $3 \times 4 - 2 = 10$ 1x1 squares, $2 \times 3 - 2 = 4$ 2x2 squares, and $1 \times 2 - 1 = 1$ 3x3 square, for a total of $10 + 4 + 1 = 15$.

31. What is the largest number of regions into which three pairs of perpendicular lines can divide a plane?

The perpendicularity doesn't actually affect the number of regions that can be created; the zeroth line creates one region, the first line adds one more region, the second line adds two more regions, the third line adds three more regions, etc. Thus, six lines can create $1 + 1 + 2 + 3 + 4 + 5 + 6 = 22$ regions.

32. A regular polygon has vertices lettered sequentially around its circumference: A, B, C, … If $\overline{FN}$ would bisect the figure, what line segment connecting vertex J to another vertex would also bisect the figure?

F to N is 8 sides, so there must be sixteen sides total. Thus, J will be across from either $J + 8 = R$ or $J - 8 = B$. R is beyond 16, so the answer is $\overline{JB}$.

33. What is the length of the latus rectum of the parabola with equation $= 3x^2 - 4x$?

The length of the latus rectum in a parabola is four times the focal distance, which for this parabola satisfies $\frac{1}{4p} = 3$. Thus, $p = \frac{1}{12}$ and thus the focal distance is $\frac{1}{12} \times 4 = \frac{1}{3}$.

34. If \$100 is invested at $10ln8$ percent annual interest compounded continuously, how much money, in dollars rounded to the nearest dollar, will be in the account after five years?

$$T = Pe^{rt} = 100e^{\frac{10ln8}{100}\times 5} = 100e^{\frac{ln8}{2}} = 100 \times 8^{\frac{1}{2}} = 100 \times 2\sqrt{2} = 200\sqrt{2} \sim 200 \times 1.41$$
$$= 200\sqrt{2} \sim 200 \times 1.414 = 282.8 \sim 283$$

35. Evaluate: $\log_{81} 27\sqrt{3}$

$$\frac{\ln 27\sqrt{3}}{\ln 81} = \frac{\frac{7}{2}\ln 3}{4\ln 3} = \frac{7}{8}$$

36. If $a(b) = 2 \times 3^{4b} - 5$, evaluate $a^{-1}\left(-\frac{43}{9}\right)$.

We can write $-\frac{43}{9} = 2 \times 3^{4b} - 5$, which becomes $\frac{2}{9} = 2 \times 3^{4b}$, then $\frac{1}{9} = 3^{4b}$, so that $4b = -2$ and $b = -\frac{1}{2}$.

37. If $c(d) = 9d + 8$ and $f(g) = g^2 - g$, evaluate $f\big(c(-2)\big)$.

$c(-2) = 9(-2) + 8 = -18 + 8 = -10$, and $f(-10) = (-10)^2 - (-10) = 100 + 10 = 110$.

38. *h* varies jointly as the square of *k* and inverse of *j*. If $h = 36$ when $j = 24$ and $k = 12$, what will *h* be when $j = 6$ and $k = 6$?

Because *k* was multiplied by $\frac{6}{12} = \frac{1}{2}$, *h* will be multiplied by $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$. Similarly, because *j* was multiplied by $\frac{6}{24} = \frac{1}{4}$, *h* will be multiplied by 4. Thus, *h* is now $36 \times \frac{1}{4} \times 4 = 36$.

39. Smithium has a half-life of 20 minutes. How many grams of a 100 kg sample of Smithium will remain after two hours, as a decimal?

Two hours is 120 minutes, which is six half-lives, and 100 kg is 100,000 g. Thus, there will be $\frac{100000}{2^6} = \frac{10^5}{2^6} = \frac{5^5}{2} = \frac{3125}{2} = 1562.5$.

40. What is the product of the roots of $m^5 - 2m^4 + 3m^2 = 4$?

Rewriting it as $m^5 - 2m^4 + 3m^2 - 4 = 0$, the product of the roots will be $(-1)^5 \frac{z}{a} = -\frac{-4}{1} = 4$.

41. How many multiples of 18 are factors of 1782?

$18 = 2 \times 3^2$ and $1782 = 2 \times 3^4 \times 11$, so the numbers we're looking for must have a single factor of two (1 choice), from two to four factors of three (3 choices), and from zero to one facto of eleven (2 choices), for a total of $1 \times 3 \times 2 = 6$.

42. Evaluate: $\prod_{f=2}^{34}\left(1-\frac{1}{f}\right)^2$

$$\prod_{f=2}^{34}\left(1-\frac{1}{f}\right)^2 = \left(1-\frac{1}{2}\right)^2\left(1-\frac{1}{3}\right)^2\left(1-\frac{1}{4}\right)^2\cdots\left(1-\frac{1}{32}\right)^2\left(1-\frac{1}{33}\right)^2\left(1-\frac{1}{34}\right)^2$$
$$= \left(\frac{1}{2}\right)^2\left(\frac{2}{3}\right)^2\left(\frac{3}{4}\right)^2\cdots\left(\frac{31}{32}\right)^2\left(\frac{32}{33}\right)^2\left(\frac{33}{34}\right)^2 = \frac{1}{34^2} = \frac{1}{1156}$$

43. What is the total number of rectangles of any size on a checkerboard? Hint: there are eight rows and eight columns on a checkerboard.

The two vertical sides of the rectangle must be chosen from nine possible lines, for a total of $9c2 = \frac{9\times 8}{2} = 9 \times 4 = 36$ options. The horizontal sides are similar, for an answer of $36^2 = 1296$.

44. When six fair coins are flipped, what is the probability that there are more heads than tails?

There are many cases where there are more heads than tails balanced by other cases where there are more tails than heads. This might make you answer $\frac{1}{2}$, but we need to consider that the case of three heads and three tails is neither more heads nor more tails. This case has a probability of $\binom{6}{3}\left(\frac{1}{2}\right)^6 = \frac{6\times5\times4}{3\times2\times2^6} = \frac{5}{2^4} = \frac{5}{16}$, leaving $\frac{11}{16}$ for the other cases, so that our answer is $\frac{1}{2} \times \frac{11}{16} = \frac{11}{32}$.

45. What is the equation of the plane through the point $(1, -2, -3)$ and perpendicular to the line through the points $(2, 3, -4)$ and $(-5, 1, -3)$? Please write your answer in the form $Ax + By + Cz = D$, where A is positive and A, B, C, and D are integers that do not collectively share a common factor.

The vector through the given points is $\langle -7, -2, 1\rangle$, so the equation of a plane perpendicular to it will be $7x + 2y - z = A$. For the given point, this becomes $7 \times 1 + 2(-2) - (-3) = 7 - 4 + 3 = 6$, for an answer of $7x + 2y - z = 6$.

46. Set N is the set of positive two-digit integers the sum of whose digits is nine. Set P is the set of positive two-digit integers that are multiples of six. How many subsets of Set N are also subsets of Set P?

Numbers that can be in a subset of both N & P must be elements of both N & P. This means they must be a multiple of both 9 and 6,which means a multiple of their LCM, 18. There are five such numbers from 18 to 90, and each of these numbers can either be in or out of a particular subset, for a total of $2^5 = 32$ possible subsets.

47. Set Q is the set of prime numbers between 0 and 10, and Set R is the set of prime numbers between 30 and 40. How many distinguishable functions have Set Q as their domain and some subset of Set R as their range?

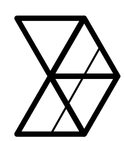
Set Q has four elements: 2, 3, 5, and 7. Set R has two elements: 31 and 37. For each element in Set Q, they can map to either 31 or 37 (two choices), so there are $2^4 = 16$ possible mappings.

48. Willa is taller than both Vince and Umberto, Tom is shorter than Sylvia, and Vince is taller than Sylvia. If they line up from shortest to tallest, how many orders might be possible?

We can determine that the heights are $W > V > S > T$, with $U < W$, so everything is determined except that there are four places that U could be, for an answer of 4.

49. An ant is at the midpoint of an edge of a regular octahedron and wishes to reach the midpoint of the opposite edge. If an edge of the octahedron measures 12 m, what is the shortest distance, in meters, that the ant can walk?

If the "top" of the octahedron is unfolded as shown to the right, the shortest distance crosses three faces and has a length that is the average of 12 and $2 \times 12 = 24$, 18.



50. What is the area, in square meters, of a triangle with sides measuring 8 m, 13 m, and 15 m?

This came up in an earlier problem… $A = \sqrt{s(s-a)(s-b)(s-c)}$, where s is half the perimeter. $\sqrt{18(10)(5)(3)} = 3 \times 5\sqrt{6 \times 2} = 15 \times 2\sqrt{3} = 30\sqrt{3}$.

51. What is the area, in square meters, of a triangle with two sides measuring 6 m and 8 m and the angle between them measuring 15°?

$$\frac{1}{2}ab\sin C = \frac{1}{2} \times 6 \times 8 \times \sin 15° = 24 \times \frac{\sqrt{6}-\sqrt{2}}{4} = 6\sqrt{6} - 6\sqrt{2}$$

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52. Evaluate in radians: $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

"The angle whose cosine is *x*" is a great way to read $\cos^{-1}(x)$. Specifically, we want the simplest one, which for cosine is defined to be between 0 and π. If you know your unit circle, this easily becomes $\frac{5\pi}{6}$.

53. If $b(c) = \frac{6c^2+5c-4}{2c-3}$, evaluate $b'(-2)$.

$b'(c) = \frac{(2c-3)(12c+5)-(6c^2+5c-4)(2)}{(2c-3)^2}$, so
$b'(-2) = \frac{(-4-3)(-24+5)-(24-10-4)(2)}{(-4-3)^2} = \frac{-7(-19)-10\times 2}{(-7)^2} = \frac{133-20}{49} = \frac{113}{49}$.

54. If $y = \begin{cases} 3x+2 & x < 1 \\ x^2+Ax+B & x \geq 1 \end{cases}$, what ordered pair (A, B) will make the function differentiable for all values of *x*?

To be differentiable, the two sub-functions must have the same value and derivative at $x = 1$. The top sub-function has a value of $3 \times 1 + 2 = 3 + 2 = 5$ and a derivative of 3 at $x = 1$. The lower sub-function has a value of $1 + A + B$ and a derivative of $2 + A$ at $x = 1$. The derivative requires that $2 + A = 3$, giving $A = 1$, while the value requires that $1 + A + B = 5$, which becomes $2 + B = 5$, giving $B = 3$.

55. What is the equation, in the slope-intercept form, of the line tangent to the graph of $y = x^3 - x + 4$ at the point $(-1, 4)$?

The derivative is $\frac{dy}{dx} = 3x^2 - 1$, which is $3(-1)^2 - 1 = 3 - 1 = 2$ at $x = -1$, so the tangent line is of the form $y = 2x + b$. Substituting the point gives $4 = 2(-1) + b$, giving $b = 6$ for an answer of $y = 2x + 6$.

56. Evaluate: $1452048 \div 312$

You just have to be careful with the long division; there are 4 312's in 1452, etc., giving an answer of 4654.

57. How many teaspoons are in seven gallons?

There are three teaspoons in a tablespoon, sixteen tablespoons in a cup, two cups in a pint, two pints in a quart, and four quarts in a gallon, for $3 \times 16 \times 2 \times 2 \times 4 = 3 \times 256 = 768$. In seven gallons, there would be $7 \times 768 = 5376$ teaspoons.

58. Round to the nearest hundred: $\frac{12345+6789}{987-98}$

This is roughly $\frac{19000}{900} \sim 20$, which rounds to 0 to the nearest hundred.

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59. Simplify by rationalizing the denominator: $\frac{162}{9+3\sqrt{7}}$

$$\frac{162}{9+3\sqrt{7}} \times \frac{9-3\sqrt{7}}{9-3\sqrt{7}} = \frac{162(9-3\sqrt{7})}{81-63} = \frac{162(9-3\sqrt{7})}{18} = \frac{18(9-3\sqrt{7})}{2} = 9(9-3\sqrt{7})$$
$$= 81 - 27\sqrt{7}$$

60. What ordered pair (j, k) satisfies the system of equations $11j - 7k = 5$ and $3j + 2k = 17$?

Adding twice the first equation to seven times the second gives $43j = 129$, which yields $j = 3$. Substituting this into the second equation gives $3 \times 3 + 2k = 17$, then $2k = 8$, so that $k = 4$.

61. If Eric can build a house in 24 days and Tom can build one in 36 days, how many days would it take them to build a house if they worked together?

Their rates add together, not their times, so their combined rate would be $\frac{1}{24} + \frac{1}{36} = \frac{3+2}{72} = \frac{5}{72}$ houses per day, so that it would take $\frac{72}{5}$ days to build one house.

62. If nine chickens can lay 24 eggs in three days, how many days would it take seventeen chickens to lay 272 eggs?

The number of chickens was multiplied by $\frac{17}{9}$, so the number of days will be multiplied by $\frac{9}{17}$. The number of eggs was multiplied by $\frac{272}{24} = \frac{68}{6} = \frac{34}{3}$, so the number of days will also be multiplied by $\frac{34}{3}$. Thus the number of days will be $3 \times \frac{9}{17} \times \frac{34}{3} = 3 \times 3 \times 2 = 18$.

63. If Li travels at 40 kmph for half the distance she intends to travel, but wants to average 50 kmph for the entire trip, what speed (in kmph) should she average for the second half of her trip?

She wants to travel $2d$ in $\frac{2d}{50} = \frac{d}{25}$ hours, but she's already been driving for $\frac{d}{40}$ hours, so she can only take $\frac{d}{25} - \frac{d}{40} = \frac{8d-5d}{200} = \frac{3d}{200}$ more hours for the second half of her trip. To go another *d* she must travel at a rate of $\frac{200}{3}$ kmph.

64. What is the distance between the *x*- and *y*-intercepts of the line $2x + 3y = 45$?

The intercepts are $\left(\frac{45}{2}, 0\right)$ and $(0,15)$, for a distance of $\sqrt{\left(\frac{45}{2}\right)^2 + 15^2} = 15\sqrt{\frac{9}{4} + 1} = 15\sqrt{\frac{13}{4}} = \frac{15\sqrt{13}}{2}$.

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65. A cowboy is at the point $(5,-3)$ and wishes to ride to the river (represented by the line $y=19-x$) before heading to town at $(-2,11)$. What is the shortest distance he can ride?

The easiest way to do problems of this sort is to reflect one of the points across the line, so that the shortest distance is just a straight line. $(5,-3)$ reflects to $(22,14)$, which is $\sqrt{24^2+3^2}=3\sqrt{8^2+1^2}=3\sqrt{65}$.

66. When Mr. Brown writes a quadratic of the form $m^2+Nm+P=0$ on the board, Quynh writes the wrong value of N and gets roots of 4 and -7, while Rowan writes the wrong value of P and gets roots of -2 and -3. What are the roots of Mr. Brown's original equation?

Quynh has the right value of P, which must be $4(-7)=-28$, and Rowan has the right value of N, which must be $-\big(-2+(-3)\big)=-(-5)=5$, for an original quadratic of $m^2+5m-28=0$ with roots of $m=\frac{-5\pm\sqrt{25+112}}{2}=\frac{-5\pm\sqrt{137}}{2}$.

67. A rectangular photograph with a perimeter of 48 cm and an area of 128 cm^2 is surrounded by a rectangular frame with each of its edges exactly 3 cm from an edge of the photograph. What is the area, in square centimeters, of just the frame?

The semi-perimeter is 24, which ends up giving lengths of 16 and 8. The frame is therefore 22 by 14 with an area of $22\times14-16\times8=308-128=180$.

68. A group of coworkers pooled their money to buy BiggiBillions lottery tickets, and ended up winning! Of course they split the winnings equally. If there had been one more coworker involved each of them would have received thirty million dollars less, while if there had been two fewer coworkers involved each of them would have received eighty million dollars more. What was the size of the jackpot, in billions of dollars expressed as a decimal?

If there are *n* people and each got a share of *s* (in millions), we can write $ns=(n+1)(s-30)=(n-2)(s+80)$. FOILing and subtracting the *ns* gives $s-30n=30$ and $-2s+80n=160$. Doubling the first and adding the second gives $20n=220$, so that $n=11$ and $s=360$, for a jackpot of $11\times360=3960$ million, which is 3.96 billion.

69. If $6stu=-20$, $s+2t+3u=5$, and $2st+6tu+3su=-4$, and $s>t>u$, evaluate $s+t+u$.

Consider the polynomial that factors to $(x-s)(x-2t)(x-3u)=0$ and expands to $x^3-(s+2t+3u)x^2+(2st+6tu+3su)x-6stu=0$. Substituting the given values produces $x^3-5x^2-4x+20=0$, with roots of -2, 2, and 5, which are *s*, 2*t*, and 3*u* in some order. The negative value must correspond to *u*, the smallest variable, so that $u=-\frac{2}{3}$. *S* must then be 5 to be the largest, leaving $t=\frac{2}{2}=1$, for a sum of $5+1-\frac{2}{3}=6-\frac{2}{3}=\frac{18-2}{3}=\frac{16}{3}$.

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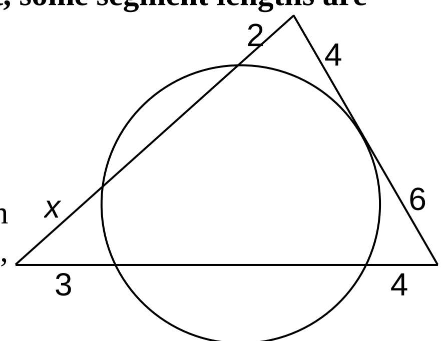
70. Seven years ago, Valerie was eight years younger than Willa will be when Xavier is 11. In six years, Willa will be three times as old as Xavier and five years older than Valerie is currently. What is the sum of their current ages?

The language implies that Xavier is younger than 11 right now. If he is X right now, he will be 11 in $11 - X$ years, so that we can say $V - 7 = W + (11 - X) - 8$. We can also write $W + 6 = 3(X + 6)$ and $W + 6 = V + 5$. Working backwards, we can say that $W = V - 1$, $V = 3X + 13$ (so $W = 3X + 12$), and $3X + 6 = 3X + 12 + (11 - X) - 8 = 2X + 15$, so that $X = 9$, $W = 39$, and $V = 40$, for a sum of 88.

71. What is the maximum possible volume, in cubic meters, of a right square pyramid with edges measuring 3 m and 5 m?

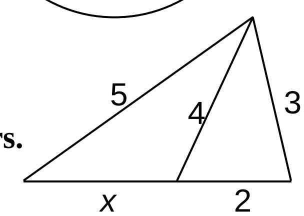
The base edges cannot be 5's, as the diagonal would be $5\sqrt{2}\sim 7$, which is more than twice 3. So, the base edges must be 3, leaving the slant edges to be the 5's. The height of the pyramid would then be $\sqrt{5^2 - \left(\frac{3\sqrt{2}}{2}\right)^2} = \sqrt{25 - \frac{9}{2}} = \sqrt{\frac{41}{2}} = \frac{\sqrt{82}}{2}$, so that the volume would be $\frac{1}{3} \times 3^2 \times \frac{\sqrt{82}}{2} = \frac{3\sqrt{82}}{2}$.

72. In the intersecting circle and triangle shown to the right, some segment lengths are given in meters. What is the value of *x*?



The rightmost vertex and surroundings gives the relationship $6^2 = 4(z + 4)$, so that $z + 4 = 9$ and thus z (the lower chord) is 5. Similarly, we can write $4^2 = 2(y + 2)$, becoming $y + 2 = 8$ and finally $y = 6$. Finally, we can write $3(3 + 5) = x(x + 6)$, which becomes $24 = x^2 + 6x$, then $x^2 + 6x - 24 = 0$ with (positive) solution $x = \frac{-6+\sqrt{6^2-4\times 1(-24)}}{2} = \frac{-6+\sqrt{36+96}}{2} = \frac{-6+\sqrt{132}}{2} = \frac{-6+2\sqrt{33}}{2} = -3 + \sqrt{33}$.

73. In the triangle to the right, all segment lengths are given in meters. What is the value of *x*?



Stewart's Theorem gives $5^2 \times 2 + 3^2 \times x = 4^2(x + 2) + 2x(x + 2)$, which becomes $50 + 9x = 16x + 32 + 2x^2 + 4x$, then $2x^2 + 11x - 18 = 0$, with (positive) solution $x = \frac{-11+\sqrt{11^2-4\times 2(-18)}}{2\times 2} = \frac{-11+\sqrt{121+144}}{4} = \frac{-11+\sqrt{265}}{4}$.

74. Three congruent beach balls with radii of 30 cm are all touching one another while sitting on the floor. Resting on top of the three is a smaller ball with a radius of 15 cm. What is the vertical distance, in centimeters, from the top of this ball to the floor?

Looking down from above, the pyramid formed by the centers of the balls is an equilateral triangle with sides of $30+30=60$ and thus a center $20\sqrt{3}$ from each vertex. The slant heights of the pyramid are $30+15=45$, so the height of the pyramid is $\sqrt{45^2-\left(20\sqrt{3}\right)^2}=5\sqrt{9^2-\left(4\sqrt{3}\right)^2}=5\sqrt{81-48}=5\sqrt{33}$. This makes the top of the upper ball $30+5\sqrt{33}+15=45+5\sqrt{33}$ above the floor.

75. What are the coordinates, in the form (x,y), of the rightmost focus of the hyperbola $9x^2-16y^2+54x+32y=79$?

Completing the square gives $9(x+3)^2-16(y-1)^2=79+9\times 3^2-16\times 1^2=79+81-16=144$, for a standard form of $\frac{(x+3)^2}{16}-\frac{(y-1)^2}{9}=1$. This hyperbola has a center at $(-3,1)$, opens to the left and right, and has a focal distance of $\sqrt{16+9}=\sqrt{25}=5$, so that the rightmost focus will be at $(-3+5,1)=(2,1)$.

76. What is the largest real value of *n* for which there is a real value of *p* satisfying $3n^2+4p^2-5np+n=6$?

As a quadratic in terms of *p*, we can write $4p^2-5np+(3n^2+n-6)=0$. To have a real value for *p*, the discriminant must be non-negative, so $25n^2-4\times 4(3n^2+n-6)\geq 0$, which becomes $-23n^2-16n+96\geq 0$, which is a downward-pointing quadratic, so that there is a greatest value of n corresponding to one of the "$=0$" cases. That root is $\frac{16-\sqrt{16^2-4(-23)96}}{2(-23)}=\frac{16-8\sqrt{2^2-(-23)6}}{2(-23)}=\frac{8-4\sqrt{4+138}}{-23}=\frac{-8+4\sqrt{142}}{23}$.

77. What is the area of a triangle with vertices at the focus and x-intercepts of the parabola $y=3x^2-10x-8$?

The axis of symmetry is $x=-\frac{b}{2a}=-\frac{-10}{2\times 3}=\frac{5}{3}$, and the y-coordinate of the vertex is $y=3\left(\frac{5}{3}\right)^2-10\left(\frac{5}{3}\right)-8=\frac{25}{3}-\frac{50}{3}-8=-\frac{49}{3}$. The focal distance satisfies $\frac{1}{4p}=3$, so that $4p=\frac{1}{3}$ and $p=\frac{1}{12}$, so the focus is at $\left(\frac{5}{3},-\frac{49}{3}+\frac{1}{12}\right)=(\frac{5}{3},-\frac{195}{12}=-\frac{65}{4})$. The x-intercepts are at $x=\frac{10\pm\sqrt{10^2-4\times 3(-8)}}{2\times 3}=\frac{5\pm\sqrt{5^2-3(-8)}}{3}=\frac{5\pm\sqrt{49}}{3}=\frac{5\pm 7}{3}$, which is 4 and $-\frac{2}{3}$. Thus, we can consider the base to be $4+\frac{2}{3}=\frac{14}{3}$ and the height to be $\frac{65}{4}$, for an area of $\frac{1}{2}\times\frac{14}{3}\times\frac{65}{4}=\frac{7}{3}\times\frac{65}{4}=\frac{455}{12}$.

78. Evaluate as a base 12 number, where the digit A represents 10 and the digit B represents 11: $B9A_{12}\times AB8_{12}$

If you can carry 12's instead of 10's, it's easiest to keep this in base 12. $8_{12}\times B9A_{12}=7A68_{12}$ after carries of 6 & 6. $B_{12}\times B9A_{12}=AA02_{12}$ after carries of 9 & 9. $A_{12}\times B9A_{12}=9A24_{12}$ after carries of 8 & 8. This makes our answer $7A68_{12}+AA020_{12}+9A2400_{12}=A98288_{12}$.

79. How many positive integers are factors of 4998?

$4998 = 2 \times 2499 = 2 \times 3 \times 833 = 2 \times 3 \times 7 \times 119 = 2 \times 3 \times 7^2 \times 17$, so there are $2 \times 2 \times 3 \times 2 = 24$ factors.

80. Arithmetic sequence C has a first term of 2109 and a common difference of 1350, and arithmetic sequence D has a first term of 3456 and a common difference of 8262. What is the smallest possible positive difference between a term of sequence C and a term of sequence D?

$1350 = 2 \times 5^2 \times 3^3$, while $8262 = 2 \times 3^2 \times 459 = 2 \times 3^4 \times 51 = 2 \times 3^5 \times 17$. These have a greatest common factor of $2 \times 3^3 = 54$, which means they can only change the separation of two terms by a multiple of 54. The initial separation of terms was $3456 - 2109 = 1347$. $1347 \div 54 = 24r51$, which means the two terms could differ by $54 - 51 = 3$.

81. What is the 10,000th smallest positive palindromic integer?

There are 9 one-digit palindromes & 9 two-digit palindromes, for a total of 18. Similarly, there are 180 three- & four-digit palindromes, and 1800 five- & six-digit palindromes, for a total of 1998 palindromes so far. There are 1000 seven-digit palindromes starting with 1, another 1000 starting with 2, etc., so that after those starting with 8 we have a total of 9998 palindromes. The 9999th palindrome is 9000009, and the 10,000th is 9001009.

82. What is the sum of the multiples of seven between 699 and 7778?

We're adding the 7's from 700 to 7777, which should be 7 times the sum from 100 to 1111, which is $7 \times \frac{1111 \times 1112 - 99 \times 100}{2} = 7(1{,}111 \times 556 - 99 \times 50) = 7(617{,}716 - 4{,}950) = 7 \times 612{,}776 = 4{,}289{,}362$.

83. Sequence A is an arithmetic sequence with first term 114 and common difference 4 and Sequence B is a geometric sequence with first term 3072 and common ratio $\frac{3}{2}$. What is the sum of the terms of Sequence A that are also in Sequence B?

Sequence A has all even numbers that are not multiples of 4 greater than 114. Sequence B will have exactly one number that is a multiple of 2 but not 4, so there will be one term in common and that will be the desired sum. $3072 = 3 \times 1024 = 3 \times 2^{10}$, so we need the term $3 \times 2^{10} \times \left(\frac{3}{2}\right)^9 = 3^{10} \times 2 = 243^2 \times 2 = 243 \times 486 = 118{,}098$.

84. When Giana is dealt three cards from a standard 52-card deck, she tells you that she does not have three of the same suit. What is the probability that she has three of the same rank?

There are $4c1 \times 13c3$ hands we're sure she doesn't have, but none of them are three of a kind, so the probability is $\frac{13c1\times 4c3}{52c3-4c1\times 13c3} = \frac{13\times 4\times 3\times 2}{52\times 51\times 50-4\times 13\times 12\times 11} = \frac{1}{17\times 25-2\times 11} = \frac{1}{425-22} = \frac{1}{403}$.

85. In how many ways can five nickels and ten dimes be arranged in a line if no two nickels may be adjacent to one another?

Put all the dimes out first. Now there are 11 spots (the ends count), that could hold at most one nickel. The answer is then $11c5 = \frac{11\times 10\times 9\times 8\times 7}{5\times 4\times 3\times 2} = 11\times 3\times 2\times 7 = 462$.

86. Helga's High School has 314 students. 37 students from the Math Club are also in Glee Club, 23 Glee Club students are also in Key Club, and 19 Key club members are also in Wii Club. No student is in exactly three of the clubs, and four times as many students are in all four of the clubs as are in exactly one of the clubs. If there are more students in Math Club than Glee Club, more in Glee Club than Key Club, and more in Key Club than Wii Club, what is the minimum number of students in Math Club if every student is in at least one of these four clubs?

The number of people in all four clubs is either 0, 4, 8, 12, or 16, leaving 0, 1, 2, 3, or 4 distributed in exactly one club, and 235, 242, 249, 256, or 263 distributed between the remaining double-memberships. If we have X extra in Math & Key, Y extra in Math & Wii, and $235 - X - Y$ (through $263 - X - Y$) in Glee & Wii, then not counting the single-clubbers we have $37 + X + Y$ in Math, $295 - X - Y$ (through $307 - X - Y$) in Glee, $42 + X$ (through $26 + X$) in Key, and $254 - X$ (through $282 - X$) in Wii. Across these cases, the sum of Math & Glee goes from 332 to 344, allowing the lowest possible Math to range from 167 to 173, and allocating any of the single-clubbers to Math doesn't help; they can go to Key & Wii and keep the Math numbers the same. Thus, the answer is 167.

87. In an unusual game, two players take turns rolling a die with colored faces. The numbers 1, 2, 3, and 4 are colored red and the numbers 5 and 6 are colored blue. The first player wins if he rolls a 1 on his first roll. Otherwise, a player wins if they roll a number with the same color as the number just rolled by the other player. What is the probability that the first player wins?

We can write $\left(\frac{1}{6}\right) + \left(\frac{3}{6}\times\frac{2}{6}\times\frac{2}{6}+\frac{2}{6}\times\frac{4}{6}\times\frac{4}{6}\right) + \left(\frac{3}{6}\times\frac{2}{6}\times\frac{4}{6}\times\frac{2}{6}\times\frac{2}{6}+\frac{2}{6}\times\frac{4}{6}\times\frac{2}{6}\times\frac{4}{6}\times\frac{4}{6}\right) + \cdots$, which after the first term is a geometric sequence with common ratio $\frac{2}{6}\times\frac{4}{6} = \frac{1}{3}\times\frac{2}{3} = \frac{2}{9}$, for an answer of $\left(\frac{1}{6}\right) + \frac{\left(\frac{3}{6}\times\frac{2}{6}\times\frac{2}{6}+\frac{2}{6}\times\frac{4}{6}\times\frac{4}{6}\right)}{1-\frac{2}{9}} = \frac{1}{6} + \frac{12+32}{216}\times\frac{9}{7} = \frac{1}{6} + \frac{44}{24\times 7} = \frac{7+11}{6\times 7} = \frac{18}{42} = \frac{3}{7}$.

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Solutions

88. Ian's boss Jenny has an unusual way of paying him. In theory, Ian's salary is $500 a week paid every week, but instead each Friday Jenny flips a coin three times. The first flip can either double or halve Ian's paycheck. The last two flips work together: If they're both heads, Jenny deducts $100 from Ian's paycheck; if they're both tails Jenny deducts $300 from Ian's paycheck; if they're a head and a tail Jenny adds $500 to Ian's paycheck. If the final result is negative, James is paid nothing, but doesn't have to pay Jenny for the privilege of working for her. What is the expected value of Ian's paycheck in dollars rounded to the nearest hundredth (cent)?

There are eight possible cases with equal probabilities: $1000 - 100 = 900$, $1000 + 500 = 1500$, 1500 again, $1000 - 300 = 700$, $250 - 100 = 150$, $250 + 500 = 750$, 750 again, and $250 - 300 = -50 \to 0$. The expected value is the sum divided by eight, $\frac{6250}{8} = 781.25$.

89. What is the area of the triangle with vertices at the points $(\mathbf{1}, \mathbf{2}, \mathbf{3})$, $(-\mathbf{2}, \mathbf{3}, -\mathbf{1})$, and $(\mathbf{4}, -\mathbf{3}, \mathbf{0})$?

The area of a triangle can be $\frac{1}{2}ab\sin C$, which is half the magnitude of the cross product of two vectors. Two vectors from the first point along the adjacent sides are $\langle -3,1,-4\rangle$ and $\langle 3,-5-3\rangle$, their cross product is $\langle -23,-21,12\rangle$, and its magnitude is $\sqrt{23^2 + 21^2 + 12^2} = \sqrt{529 + 441 + 144} = \sqrt{1114}$, so that the area of the triangle is $\frac{\sqrt{1114}}{2}$.

90. In a set of nine integer test scores from 0 to 100 inclusive, the range is 71, the unique mode is 69, and the mean is 65. What is the smallest possible value of the median?

For a low median (with a fixed mean), we want all the other scores to be as high as possible, so we can write 29, $m - 3$, $m - 2$, $m - 1$, m, 69, 69, 99, 100, then the sum $4m + 360 = 9 \times 65 = 585$, which becomes $4m = 225$ and gives $m = 56.25$, for an answer of 57. There are cases with more 69's to consider, but they will give higher medians.

91. The data set $\{\mathbf{5}, \mathbf{10}, \mathbf{11}, \mathbf{14}, \mathbf{18}, \mathbf{20}\}$ has three other positive integer elements, *x*, *y*, and *z*. If the unique mode is less than the mean, which in turn is less than the median, what is the largest possible sum of *x*, *y*, and *z*?

The median must be between 10 and 18. For the median to be higher than the mode, the median must be between 10 and 14. For the median to be 14 (which allows the highest mean, and thus the highest sum), there must be one of x, y, and z lower than 14 and two higher, with the lower one being 5, 10, or 11 to create a lower mode. The sum of *x*, *y*, and *z* can be as large as $9 \times 14 - 1 - 78 = 126 - 79 = 47$.

92. Set L is the set of positive three-digit integers whose units digit is smaller than their tens digit, which in turn is smaller than their hundreds digit. Set M is the set of positive three-digit integers whose tens digit is equal to the sum of their units and hundreds digits. How many elements are in the set $L' \cap M$?

We need numbers that are in Set M but not in Set L, and conveniently all of the elements of M are not in L, because all of the elements of M have tens digits that are greater than or equal to their hundreds digit. Thus, our answer is simply the number of elements in M. For a hundred's digit of one, there are 9 elements in M from 110 to 198. For two, there are 8 from 220 to 297. Continuing, our answer will be the sum of the numbers from 9 to 1, which is 45.

93. The heist of the century has taken place, and all of the criminal masterminds have been rounded up. We know that exactly two people committed the crime, and we're sure that those two are among the suspects on hand. Each suspect was questioned separately and made the two statements below. Our psychologist assures us that in an attempt to throw us off, all suspects will be sure to make one true statement and one false statement. Which two criminals committed this particular crime? "I" refers to the speaker.

S: I didn't do it! W & V did it.
T: OK, I did it. U & S did it.
U: V did it. Neither X nor I did it.
V: X did it. T did it but X didn't.
W: T didn't do it. At least one of S & T did it.
X: W & I did it. At most one of U & W did it.

Each person either did True-then-False or False-then-True. S must have done True-then-False, so S didn't do it, and it wasn't W&V together. T must be True-then-False, so T did it with someone else. Because S didn't do it and T did, there are four possible teams: T&U, T&V, T&W, and T&X. If it was T&U, U would be False-then-False, which is not allowed. If it was T&V, U would be True-then-True, which is also not allowed. If it was T&W, the statement pairs are T/F, F/T, F/T, F/T, and F/T, which looks like a solution! If it was T&X, U would be False-then-False, which is not allowed. Thus, the answer is T&W.

94. If $\tan a = \frac{3}{4}$ and $\frac{\pi}{2} < a < \frac{3\pi}{2}$, evaluate $\tan\left(\frac{a}{2}\right)$.

Because $\tan a > 0$, $\pi < a < \frac{3\pi}{2}$, and $\frac{\pi}{2} < \frac{a}{2} < \frac{3\pi}{4}$, so that $\tan\left(\frac{a}{2}\right) < 0$. Because $\tan a = \frac{3}{4}$ and $\pi < a < \frac{3\pi}{2}$, $\sin a = -\frac{3}{5}$ and $\cos a = -\frac{4}{5}$. In general, $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$, so $\cos a = \cos^2\frac{a}{2} - \sin^2\frac{a}{2} = 2\cos^2\frac{a}{2} - 1 = 1 - 2\sin^2\frac{a}{2}$. Thus, $\sin\frac{a}{2} = \pm\sqrt{\frac{1-\cos a}{2}}$ and $\cos\frac{a}{2} = \pm\sqrt{\frac{1+\cos a}{2}}$. Because $\frac{\pi}{2} < \frac{a}{2} < \frac{3\pi}{4}$, $\sin\frac{a}{2} > 0$ and $\cos\frac{a}{2} < 0$. $\sin\frac{a}{2} = \sqrt{\frac{1-\left(-\frac{4}{5}\right)}{2}} = \sqrt{\frac{\frac{9}{5}}{2}} = \sqrt{\frac{9}{10}}$ and $\cos\frac{a}{2} = -\sqrt{\frac{1+\left(-\frac{4}{5}\right)}{2}} = -\sqrt{\frac{\frac{1}{5}}{2}} = -\sqrt{\frac{1}{10}}$, so that $\tan\frac{a}{2} = \frac{\sin\frac{a}{2}}{\cos\frac{a}{2}} = \frac{\sqrt{\frac{9}{10}}}{-\sqrt{\frac{1}{10}}} = -\sqrt{9} = -3$.

95. What value(s) of z between 0 and 2π inclusive satisfy $6\sin^2 z = 4 - 5\sin z$?

Rewriting as $6\sin^2 z + 5\sin z - 4 = 0$, we can factor to $(3\sin z + 4)(2\sin z - 1) = 0$, so that $\sin z = -\frac{4}{3}$ or $\sin z = \frac{1}{2}$. The first is impossible, but the second has solutions when $z = \frac{\pi}{6}$ or $\frac{5\pi}{6}$.

96. Simplify: $\frac{6a^5+a^4-3a^3+8a^2+18a-24}{2a^2-3a+4}$

Doing long division, the quotient must be $3a^3 + \cdots$, which would give a product of $6a^5 - 9a^4 + 12a^3$, so that the quotient must be $3a^3 + 5a^2 + \cdots$, which would give a product of $6a^5 + a^4 - 3a^3 + 20a^2 + \cdots$, so that the quotient must be $3a^3 + 5a^2 - 6$.

97. What is the area contained by the graph of the parametric equations $x = -1 - \sqrt{3}\sin t$ and $y = 1 + \sqrt{5}\cos t$?

We can relate x & y through the identity $\sin^2 t + \cos^2 t = 1$, so that $\left(\frac{x+1}{\sqrt{3}}\right)^2 + \left(\frac{y-1}{\sqrt{5}}\right)^2 = 1$, which is an ellipse with $A = \pi ab = \pi \times \sqrt{3} \times \sqrt{5} = \pi\sqrt{15}$.

98. What is the area of the region bounded by the *x*-axis and the graph of $y = 3x^2 + 5x - 2$?

Factoring gives $y = (3x-1)(x+2)$ with x-intercepts at $\frac{1}{3}$ and -2, so that the area will be $\int_{\frac{1}{3}}^{-2} 3x^2 + 5x - 2\,dx = x^3 + \frac{5}{2}x^2 - 2x\Big|_{\frac{1}{3}}^{-2} = -8 + 10 + 4 - \left(\frac{1}{27} + \frac{5}{18} - \frac{2}{3}\right) = 6 - \left(-\frac{19}{54}\right) = \frac{343}{54}$, which is of course the negative of the correct answer, because the region lies below the x-axis.

99. If $d(f) = (f+2)^{f-3}$, what is $d'(1)$?

To take the derivative, we need to first take the natural logarithm of both sides of the equation, getting $\ln\big(d(f)\big) = (f-3)\ln(f+2)$. Taking the derivative of both sides gives $\frac{d'(f)}{d(f)} = \frac{f-3}{f+2} + \ln(f+2)$, so that $d'(f) = \left(\frac{f-3}{f+2} + \ln(f+2)\right)d(f) = \left(\frac{-2}{3} + \ln 3\right) \times 3^{-2} = -\frac{2}{27} + \frac{\ln 3}{9}$.

100. Use the Trapezoid Rule with $\Delta x = 1$ to approximate $\int_2^5 (x^2+4)\,dx$.

There are three trapezoids, each with a "height" of 1 and with bases measuring 8, 13, 20, and 29. The outer bases get used in just one trapezoid each, but the inner bases are used in two trapezoids each, so that the total area will be $\frac{8+26+40+29}{2} \times 1 = \frac{103}{2}$.