2013 Team Scramble
Thursday, November 7th, 2013

This test consists of 100 problems to be solved in 30 minutes by a large team. All answers must be exact, complete, and in simplest form. To ensure consistent grading, if you get a decimal, mixed number, or ratio as any part of an answer, it should be expressed as a fraction unless otherwise specified in the problem. A correct answer to a problem scores one point; a blank or incorrect answer to a problem scores no points. All answers must be written on the answer sheet in the boxes provided; work or answers written elsewhere will not be scored.

1. Evaluate: $9754321 - 2356789$

2. Evaluate: $987654321 \div 289$

3. What number is 37 more than twice the number that is 41 less than the product of 53 and 59?

4. Evaluate **as a mixed number**: $7\frac{5}{6} \times 4\frac{8}{9}$

5. Evaluate: $-1(-2 - 3^{-2})^{-1}(-2) - (-3) - 2^{-1}$

6. How many hours are there in a leap year?

7. Evaluate: $\log_3 4 \times \log_5 27 \times \log_{256} 25$

8. Evaluate: $6 + (8 \times 9 - 4^2) \div 2$

9. Evaluate: $1234^2 - 123^2$

10. Arrange the letters in order of ascending value:
    A = the number of days in March
    B = the number of inches in a yard
    C = the number of diagonals in a decagon
    D = the number of weeks in a year

11. What value(s) of f satisfy $29f + 38 = 2329$?

12. Simplify by multiplying and combining like terms: $(2h + 3)(4h - 5)(6 - 7h)$

13. What ordered pair, in the form (j, k), satisfies the equations $4k - 3j = 56$ and $2j + 5k = 1$?

14. If Matt can eat a pizza in an hour and Tom can eat a pizza in 45 minutes, how many minutes would it take the two of them to eat a pizza together?

15. On average, three people can remodel seven bathrooms in 24 days. On average, how many people would be needed to remodel 28 bathrooms in eight days?

16. Aaron bikes 50 km at 30 km/hr, then takes another hour to ride 20 km. What was his average speed in km/hr over his entire ride?

17. In which quadrant does the point $(7, -1)$ lie?

18. What is the equation, in slope-intercept form, of the line through the point $(2,3)$ and perpendicular to the line $-3x + 5y = 987$?

2013 Team Scramble
Thursday, November 7th, 2013

19. What is the distance between the point $(-4,-9)$ and the midpoint of the line segment from the point $(4,-7)$ to the point $(-2,19)$?

20. What is the shortest distance between the point $(5,-3)$ and the line $y=-\frac{1}{7}x-8$?

21. The point $(-9,0)$ is reflected across the line $y=x+1$ to point L, which is then reflected across the line $x=-7$ to point M. What are the coordinates, in the form (x,y), of point M?

22. What are the coordinates, in the form (x,y), of the vertex of the parabola $y=2x^2-8x+31$?

23. When a positive integer's digits are reversed to form a new positive integer that does not begin with zero, the resulting number is 693 less than the original number. When all numbers satisfying this condition are written in ascending order, what number is 11th in this list?

24. A troop of Girl Scouts contributes equally to buy a kayak. If there had been one more girl in the troop, each girl's contribution would have been five dollars less. If there had been another girl beyond that one, each girl's contribution would have been another four dollars less. How much did the kayak cost?

25. If Q quarters can exactly buy R ounces of fudge, how many ounces of fudge could be purchased with D dimes?

26. If 8 Nether Engines are equivalent to 12 Oranges, 9 Picks are equivalent to 15 Quills, and 10 Nether Engines are equivalent to 18 Picks, how many Quills are equivalent to 900 Oranges?

27. I have 60 coins in my piggy bank, with each being either a nickel, a dime, or a quarter. The number of dimes is one more than twice the number of nickels, and the total value of the coins is \$10.85. How many quarters are there?

28. What value(s) of v satisfy $\frac{v+1}{2v-3}=\frac{3+4v}{5-6v}$?

29. If $w(a)=2a(3a+4)^5$, evaluate $w(-2)$.

30. What ordered triple(s), in the form (h,j,k), satisfy the equations $h+j+k=5$, $2h+j-k=12$, and $h-j+k=-9$?

31. When Mr. Plough writes a quadratic equation of the form $m^2+Nm+P=0$ on the board, Carmen miscopies the value of N, getting roots of -2 and 10. Rowan miscopies the value of P, getting roots of -7 and 8. What are the correct roots of Mr. Plough's equation?

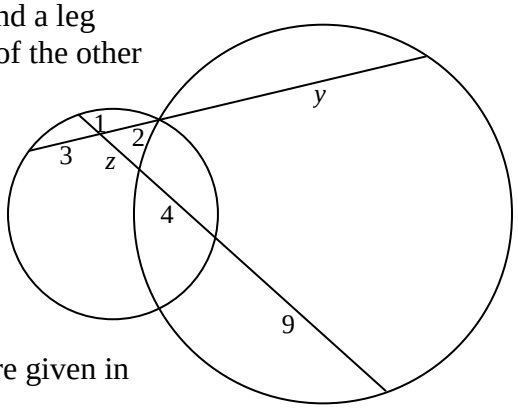
32. A rectangular picture's width is twice its height, and the picture is in a rectangular frame that extends 2 cm on all sides of the picture. If the area of only the picture frame (not including the area of the picture it contains) is 208 cm^2, what is the height of the picture in centimeters?

2013 Team Scramble
Thursday, November 7th, 2013

33. A right triangle has a hypotenuse measuring 12 cm and a leg measuring 9 cm. What is the length, in centimeters, of the other leg?

34. What is the area, in square meters, of an isosceles triangle with sides measuring 10 m and 4 m?

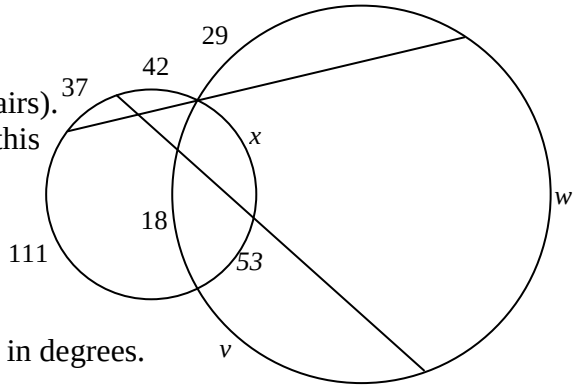
35. What is the most specific name for a triangle with no two angles congruent?



36. In the figure to the upper right, all segment lengths are given in meters. Evaluate $y + z$.

37. In a particular quadrilateral, opposing sides are congruent to one another (and there are two such pairs). What name can be given to **any** quadrilateral with this property?

38. What is the perimeter, in meters, of a $150°$ sector of a circle with a diameter of 18 m?

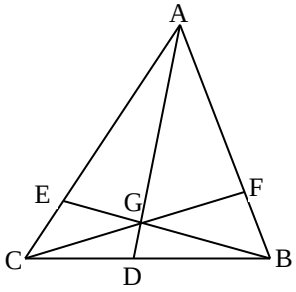


39. In the figure to the right, all given arc measures are in degrees. Evaluate $w + x$.

40. What is the area, in square meters, of an equilateral triangle that is circumscribed about a circle with a radius of 12 m?

41. What is the name of a polygon with eight sides?

42. In ΔABC to the right, cevians $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ intersect at G. If the ratio between the areas of ΔACF and ΔBCF is 3 and the ratio between the areas of ΔACD and ΔABD is $\frac{3}{2}$, what is the ratio (expressed as a fraction if it's not an integer) of the area of ΔAEB to that of ΔCEB?



43. A polyhedron has a surface area of 100 m^2 and a volume of 50 m^3. What is the volume, in cubic meters, of a similar polyhedron has a surface area of 2500 m^2?

44. What is the volume, in cubic meters, of a regular octahedron with edges measuring 8 m?

45. Two spherical balls are rolled into the same corner of a rectangular room with vertical walls, so that each touches both walls, the floor, and the other ball. What is the ratio of the radii of the larger ball to that of the smaller?

46. What is the length, in meters, of the altitude to the longest side of a triangle with sides measuring 12 m, 15 m, and 21 m?

2013 Team Scramble
Thursday, November 7th, 2013

47. A horse is tethered to outside of the 60° angle of a stable in the shape of a 30-60-90 triangle. If the hypotenuse of the stable measures 20 m and the horse's tether is 22 meters long, what is the area, in square meters, the horse can graze? Note: the horse cannot get into the stable.

48. What is the largest number of regions into which a line, a circle, and a triangle can divide a plane?

49. The complement of $\angle B$ is equal to the supplement of $\angle C$, while the supplement of $\angle B$ is $52°$ larger than the complement of $\angle D$. What is the positive difference, in degrees, between the supplements of $\angle C$ and $\angle D$?

50. Simplify in terms of $i = \sqrt{-1}$: $i(2 + 3i) - 4(5 - 6i) + (2i)^7$

51. What are the coordinates, in the form (x, y), of the center of the hyperbola with equation $3x^2 - 5y^2 + 18x - 40y = 100$?

52. What is the maximum number of points in which equations of the forms $y = AB^{Cx} + D$ and $y = Fx^3 + Gx^2 + Hx + J$ can intersect?

53. What value(s) of *k* satisfy $4^{k+1} - 5 \times 2^{k+5} + 1024 = 0$?

54. How many distinguishable functions map the set of one-digit prime numbers onto the set of positive two-digit palindromic integers?

55. Suppose that *m* is directly proportional to $\frac{n^2}{p^3}$. If $m = 216$ when $n = p = 216$, what value of *m* corresponds to $n = 108$ and $p = 72$?

56. $1000.00 is invested at 2% annual interest compounded annually. If it's left alone, how much money, in dollars rounded to the nearest hundredth (i.e. cent), will be in the account after three years?

57. What is the product of the roots of $2q^6 + 3q^4 + 5q^3 - 7q^2 - 8 = 0$?

58. What is the sum of the seven complex seventh-roots of $7 - i$?

59. Express the base-11 number 754_{11} as a base-10 number.

60. Express the base-9 number 1358_9 as a base-3 number.

61. What is the sum of the positive integer factors of 240?

62. How many positive integer factors of 6000 are multiples of 6?

63. Tammy writes the number 123 and every 235th number after that, while Ula writes the number 347 and every 450th number after that. What is the smallest difference between two of the numbers written?

2013 Team Scramble
Thursday, November 7th, 2013

64. What is the sum of the twenty smallest palindromic integers greater than 9000?

65. What is the largest prime number that is a factor of 6,531,840?

66. What is the sum of the terms of an infinite geometric sequence with first term 24 and common ratio $\frac{1}{2}$?

67. What is the third term of a harmonic sequence with first term 4 and second term 6?

68. What is the sum of the three missing terms of the sequence beginning 1, 3, 4, 6, 10, 14, 15, 21, 24, __, __, __, 45, 55, 44, …?

69. What is the 31st term of an arithmetic sequence with first term 32 and common difference 33?

70. What is the missing term of the sequence WON, TOO, FOR, __, ATE?

71. What is the sum of the 20 smallest positive perfect squares?

72. What is the sum of the two-digit positive integers that are not divisible by 7?

73. When three cards are drawn from a standard 52-card deck, what is the probability that at least two of the three are of the same rank? E.g. two Kings or three Sevens, etc.

74. When four fair six-sided dice are rolled, what is the probability that the sum of the numbers shown on their upper faces is 8?

75. A trusted friend draws two marbles from a bag containing three red, four white, and five blue marbles, and tells you that they are not both blue (either one might be blue, however). What is the probability that the two marbles are the same color?

76. In how many distinguishable ways can the letters in the word "CIRCUMFERENCE" be arranged?

77. The members of the art club were surveyed on gender (boy or girl) and favorite color (red, blue, or green), with the following results: the number of members who liked red was half the number of boys, the number of members who liked green was five times the number of boys who liked blue, the number of girls was equal to the number of boys who liked green, and the number of members who liked blue was half the number of boys who liked red. If exactly one girl liked blue, what is the fewest number of members that could be in the art club?

78. In the hot new casino game, you pay twenty dollars for a chance to flip four fair coins, after which the Floupier will count the number of tails (T) and pay you 3^T dollars. What is your expected gain if you play this game? (If you are expected to lose money, your answer would be negative.)

79. In one segment of the hit game show "That's the Price, Right?", host Honty Mall asks the contestant to point at one of five suitcases. Two suitcases are known to contain a Good Prize (and Honty knows which they are), while the others all contain Nothingness. After the contestant points, Honty opens a suitcase that he knows has Nothingness and shows it to the contestant, then offers the contestant the opportunity to change which suitcase she is pointing to. If the contestant points to a new suitcase that is neither her first choice nor the one Honty opened, what is the probability that it contains a Good Prize?

80. I draw two cards from a standard 52-card deck, look at them, and truthfully tell you that they are not of the same suit. What is the probability that I drew a pair (two cards of the same rank)?

81. If $U = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$, what is the sum of the elements of U^{-1}?

82. What is the volume of the tetrahedron with vertices at the points $(1,1,1)$, $(-2,3,-4)$, $(0,1,2)$, and $(4,3,-2)$?

83. What is the equation, in the form $Ax + By + Cz = -41$ of the plane through the points $(1,2,3)$, $(2,-3,5)$, and $(-2,4,1)$?

84. What is the mean of the mode, median, and range of the dataset $\{1, 4, 8, 13, 19, 17, 14, 10, 5, 8, 12\}$?

85. In a seven-element set of integer test scores from 0 to 100 inclusive, the mean is 69, the unique mode is 74, and the median is 70. What is the maximum possible value of the range?

86. Set V is the set of all positive three-digit integers that contain a 7. Set W is the set of all positive multiples of 12 less than 1000. How many elements are in the set $W \cup V$?

87. In the cryptarithm below, in which each instance of a letter represents the same digit and no two different letters represent the same digit (e.g. if one A is a 1, every A is a 1 and no B is a 1), what is the largest possible value of the four-digit number $ABCD$?
$$\begin{array}{r} BCD \\ +ABC \\ \hline DAB \end{array}$$

88. The numerals 1, 2, 3, 5, and 7 and the operators $+$, $-$, $\times$, and $\div$ are used exactly once each, along with any number of parentheses, to create an expression. What is the maximum possible value when such an expression is evaluated?

89. When five people sit on a bench, Victor sits somewhere to Willa's left, Xavier sits somewhere to Yessica's right, and Zane doesn't sit next to Willa. How many different seating arrangements are possible?

90. In ΔFGH, $FG = 36$ m, $m\angle F = 75°$, and $m\angle G = 30°$, what is the length, in meters, of $\overline{GH}$?

91. Evaluate: $\sec\left(\frac{119\pi}{6}\right)$

92. What is the area, in square meters, of ΔJKL if $JK = 18$ m, $KL = 36$ m, and $m\angle K = 120°$?

93. What are the coordinates, in rectangular (x, y, z) form, of the point with spherical coordinates $\left(12, \frac{\pi}{3}, \frac{5\pi}{4}\right)$, where the first angle is the azimuthal angle?

94. What is the name for the shape defined by the parametric equations $x = t^2 - 5$ and $y = 2t^2 + 1$ for real values of *t*?

95. If $m(n) = 2n(n^2 + 1)^3$, evaluate $m'(1)$.

96. A 25-foot ladder is standing on horizontal ground and propped against a vertical wall, but it's slipping so that its top is sliding down the wall at 2 ft/sec. At how many feet per second is the base of the ladder sliding away from the wall at the moment when the top of the ladder is only 7 feet off the ground?

97. What is the total area enclosed by the functions $y = x^2$ and $y = x^3$ and the lines $x = -2$ and $x = 2$?

98. What is the equation of the line tangent to the graph of $y = 3x^2 - 1$ at the point where $x = -2$?

99. What is the average value of the function $p(q) = 2q(3q^2 + 4)$ on the interval $[0,16]$?

100. A spherical amoeba's radius is increasing at a rate of 24m per second. At what rate, in square meters per second, is its surface area increasing at the moment it is 24m in diameter?