2014 Four-by-Four Competition
Thursday, January 30th, 2014

Round 1

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1. What are the coordinates, in the form (x, y), of the vertex of the parabola $x = 2y^2 - 24y + 89$?

2. What is the area, in square meters, of a right triangle with a 30° angle and a hypotenuse measuring 24 m?

3. Evaluate: $(2i)^3 - (4i)^5 + (7i)^2 - (9i - 8)$

4. When two points are chosen on a yardstick, what is the probability that they are within six inches of one another?

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Round 2

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5. What is the sum of the odd integers between 6 and 24?

6. An angle's supplement is ten times its complement. What is the angle's measure, in degrees?

7. What number is 87 less than two-thirds of the product of 327 and 18?

8. What are the coordinates, in the form (x, y), of the center of the hyperbola with equation $4x^2 - 6y^2 + 8x + 10y = 12$?

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Round 3

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9. Evaluate: $\left(\frac{81}{16}\right)^{-\frac{3}{2}}$

10. What value(s) of b satisfy $9(2b + 7) = 3(8b - 5)$?

11. In a regular polygon, the sum of the interior angles is 84 times the measure of an exterior angle. How many sides does the polygon have?

12. In a seven-element set of integer test scores from 0 to 100 inclusive, the mean is 79, the unique mode is 68, and the range is 45. What is the largest possible value of the median?

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Round 4

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13. Evaluate: $987.6 + 53.19 + 8.529 + .9810$

14. What is the area in the Cartesian Plane enclosed by the locus of points satisfying $5x^2 + 6y^2 = 7$?

15. A llama is tethered to an external corner of a hexagonal grain silo with no openings at ground level. If the hexagon has sides measuring 6 m and the llama leash is 15 m long, what is the area, in square meters, of the region in which the llama can roam?

16. What is the sum of the positive integers that are factors of 420?

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Round 5

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17. What is the period of $y = |2\sin(3x) - 4\cos(5x)|$?

18. When the magic number is increased by 357, this result is divided by 7, and this result is decreased by 197, the final result is 247. What is the magic number?

19. A $135°$ sector of a circle has an area of 294π m^2. What is its perimeter, in meters?

20. When four cards are drawn from a standard 52-card deck, what is the probability that they are four different suits and four different ranks?

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Round 6

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21. Evaluate, expressing your answer in standard form: $\frac{(8.4\times10^{3})(3.5\times10^{-1})}{(2.8\times10^{-2})(1.5\times10^{0})}$

22. How many positive five-digit integers are palindromic multiples of four?

23. What value(s) of c satisfy $24c^2 - 49c - 108 = 0$?

24. Set W is the set of positive three-digit integers containing three different digits, Set V is the set of positive three-digit integers with at least one digit that is a 3, and Set T is the set of positive three-digit integers that are multiples of three. How many elements are in the set $W \cap (V' \cup T)$?

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Round 7

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25. What are the coordinates, in the form (x, y), of the local maximum of the function $y = x^3 - 12x - 9$?

26. A shepherd has his flock at $(5,7)$ and wishes to take it to the river $(y = x - 20)$ before going to the ranch at $(0,0)$. What is the shortest total distance they can travel?

27. How many elements of a geometric sequence with first term 3 and common ratio 2 are also part of a harmonic sequence with first term 1200 and second term 1000?

28. A right rectangular prism has a length, width, and depth that are all integers when measured in meters. If the surface area of the prism is 432 m^2, what is the maximum possible volume of the prism, in cubic meters?

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Round 8

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29. Evaluate: $1098^2 - 912^2$

30. What is the remainder when 11^{111} is divided by 1000?

31. My piggy bank contains 29 coins, each of which is either a penny, nickel, dime, or quarter. The total value of all the coins is $2.86, and the number of quarters is equal to the sum of the numbers of nickels and dimes, and is less than the number of pennies. How many nickels are there?

32. When 123 people were surveyed, 94 liked sunny days, and 75 liked windy days. What is the smallest number of people who could have liked both?

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Round 9

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33. A Hailstone Sequence is a recursive sequence of positive integers in which $a_n = \frac{a_{n-1}}{2}$ if a_n is even, but $a_n = 3a_{n-1} + 1$ if a_n is odd. Choosing different values of a_1 results in different Hailstone Sequences. What is the smallest value of a_1 that will generate 11 as part of its Hailstone Sequence?

34. A convex polygon can have over 1000 diagonals drawn in it. What is the smallest number of sides the polygon could have?

35. What is the equation (in the form $Ax + By + Cz = D$ where A is positive and A, B, C, and D have a collective greatest common factor of 1) of the plane containing the point $(-2, -6, -3)$ and the line $\frac{x-4}{8} = \frac{y+5}{8} = \frac{z-1}{2}$?

36. The members of the investment club contributed evenly to buy BrownCo. If there had been one more member, each would have contributed three billion fewer dollars. If there had been four fewer members, each would have contributed fifteen billion more dollars. What was the purchase price of BrownCo, in **trillions** of dollars as a decimal?

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Round 10

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37. Ten lines are drawn in a plane. When considering all 45 possible pairs of lines, there are at least two pairs in which the lines are parallel, at least two pairs in which the lines are perpendicular, and at least two pairs in which the lines meet at an angle other than ninety degrees. What is the fewest number of regions into which the ten lines can divide the plane?

38. A triangle has angles measuring 30° and 45° and a side measuring 18 m. What is the shortest possible length, in meters, of another side of the triangle?

39. When six friends sit in six seats in a row at the movies, Reva and Quincy sit next to one another, Pam and Oly do not, Nora sits somewhere to Mark's left, and Reva sits somewhere to Pam's right. If Quincy sits on one end of the row and Oly does not, how many seating arrangements are possible?

40. What is the value of g in the solution to the system of equations $d + f + g + h + j = 11$, $d - 2f - 3g + 3h - 2j = -49$, $2d + f - h + j = 15$, and $2d - f + g - 3h - j = 68$?

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